NATIONAL EXAMINATIONS DECEMBER 2008

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumption made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. 20 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1.Consider the following differential equation:

$$(x^3 - 1)\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + xy = 0$$

Find two linearly independent power series solutions about the ordinary point x=0.

2. (a) Find the Fourier series expansion of the periodic function F(x) of period p=2π.

$$F(x) = x^2 \quad ; \qquad -\pi \le x \le \pi$$

(b) Differentiate the Fourier series expansion of F(x) to find the Fourier series of the following periodic function of period p=2π.

$$G(x) = x \quad ; \qquad -\pi < x < \pi$$

Then integrate the Fourier series expansion of F(x) to find the Fourier series expansion of the following periodic function of period p=2π.

$$H(x) = x(\pi - x)(\pi + x) \quad ; \quad -\pi \le x \le \pi$$

3. Evaluate y(1.1) using a fifth degree Taylor polynomial if:

$$\frac{dy}{dx} = x^3 - \frac{1}{y} \quad ; \quad y(1)=1$$

4.Given the following data, calculate f(-1.5) using Newton's interpolating polynomial of highest possible degree.

x	−3	−2	−1	0	1	2	3	4
f(x)	221	38	−9	−10	-7	6	83	326

5. Use the Romberg algorithm with n = 2 to find the area bounded by $f(x)=(5+3x^2)^{1/3}$, x=0 , x=2 and y=0.
Hint: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(0,0)		
R(1,0)	R(1,1)	
R(2,0)	R(2,1)	R(2,2)

where

$$R(0,0)=\frac{1}{2}(b-a)[f(a)+f(b)]$$

$$R(n,0)=\frac{1}{2}\ R(n-1,0)+h\sum_{k=1}^{2^{n-1}} f[a+(2k-1)h]$$

$$\text{where } h=\frac{b-a}{2^n}$$

$$R(n,m)=R(n,m-1)+\frac{1}{4^m-1}\ [R(n,m-1)-R(n-1,m-1)]$$

6. Consider the following linear simultaneous differential equations:

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+8y=f(x)$$

where

$$f(x)=\begin{cases}3, & 0<x<2\pi \\ 0, & 2\pi<x\end{cases}$$

$$y(0)=1 \quad ; \quad y'(0)=-1$$

7(A). One root of the equation $\tan^{-1}x + x^2 - 3 = 0$ lies between −2.2 and −2. Use the method of bisection five times to find a better approximation of this root. (Note: Carry five significant digits in your calculations).

7(B). One root of the equation $\ln(x + 3) - 5\cos x = 0$ is close to $x_0 = -1.4$. Use the following iterative formula twice to find a better approximation of this root (Note: Carry seven significant digits in your calculations).

$$x_{i+1} = x_i - \frac{f(x_i)}{f^{(1)}(x_i) - \dfrac{f(x_i)f^{(2)}(x_i)}{2f^{(1)}(x_i)}}$$

Hint: Let $f(x)=\ln(x + 3) - 5\cos x$. Note that $f^{(1)}(x)$ and $f^{(2)}(x)$ represent the first and second derivatives of f(x) respectively.

8. The symmetric positive definite matrix $A = \begin{bmatrix} 4 & 6 & -8 \\ 6 & 18 & -18 \\ -8 & -18 & 45 \end{bmatrix}$ can be written as the product of a lower triangular matrix $L=\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ and its transpose L^T, that is $A=LL^T$. Find L and then use L and L^T to solve the following system of three linear equations:

$$\begin{aligned} 4x_1 + 6x_2 - 8x_3 &= 8 \\ 6x_1 + 18x_2 - 18x_3 &= 27 \\ -8x_1 - 18x_2 + 45x_3 &= -51 \end{aligned}$$