National Exams December 2008

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.
2. This is a **CLOSED BOOK EXAM**.
3. Candidates may use one of two calculators, the Casio or Sharp **approved models**.
4. Answers to question 1 plus any **three** of questions 2 to 6 constitutes a complete exam paper.
5. Answer question 1 in the space provided on the exam paper.
6. The first three questions as they appear in the answer book will be marked.
7. Each question is of equal value. Question 1 is mandatory.

1. Answer question one in the space provided. A phrase or a diagram and a phrase is all that is required in most cases. [25 marks, one mark for each letter]

a) This exam is about light and optics. Define *geometrical optics*.

b) Define *physical optics*.

c) What problems with geometrical optics led to the development of physical optics?

d) State the *law of reflection*

e) State the *law of refraction*.

f) Complete any two rows of the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

Maxwell's equations are:

$$\nabla \cdot D = \rho_f$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J_f + \frac{\partial D}{\partial t}$$

g) to j) Beside each equation write the name of the law or the principle that each equation describes.

k) Circle the term that Maxwell contributed.

l) Over what frequency range are Maxwell's equations valid?

m) Maxwell's equations can be solved under certain approximations to find that *E* and *B* are transverse plane waves. What are the approximations?

n) Describe a transverse plane wave.

o) For most problems an analysis in terms of monochromatic plane waves is not restrictive. Why?

p) Write an equation for the *Poynting vector* and give the SI units for the quantity the Poynting vector represents.

q) What is the *refractive index of a material* defined to be equal to?

r) What defines the *direction of polarization* of light?

s) What is the difference between *interference* and *diffraction*?

t) What does the concept of *spatial coherence* deal with?

u) What does the concept of *temporal coherence* deal with?

v) Define *dispersion* as it relates to the refractive index.

w) What does the *phase velocity* of a wave equal?

x) What does the *group velocity* of a wave equal?

y) State the *principal of reversibility.*

2. Write equations for the electric field of a plane wave of light that is (i) linearly polarized; (ii) circularly polarized; and, (iii) elliptically polarized. Include the time and spatial parts of the electric field and indicate the direction of propagation of the plane wave. (5 marks)

Mica has refractive indices of 1.600 and 1.595 for propagation of light normal to the cleavage plane and polarized along the principal axes.

(b) Design an optical element using mica that acts as a quarter wave plate for red light and as a half wave plate for blue light. Draw a diagram, specify the orientation and dimensions of the mica, and explain your reasoning. (14 marks)

(c) Place the optical element, which you designed in part (b) of this question, between crossed polarizers and illuminate with white light.

Provide calculations to show how the colour that is transmitted changes as:
(i) the mica is rotated around the normal to the cleavage plane of the mica? (3 marks)
(ii) as the output polarizer is rotated around the optical axis of the system? (3 marks)

3. (a) Define far-field as it relates to optics and state how the far-field is calculated. (3 marks)

(b) Sketch the far-field of a diffraction grating that is composed of 10 lines with a spacing between lines that is twice the width of the lines in the diffraction grating. Assume that the grating is illuminated at normal incidence by a monochromatic plane wave with a wavelength equal to the spacing between the lines. (3 marks)

(c) Sketch the far-field for a blazed grating, with a blaze angle of 10 deg. Assume that the grating has 10 lines and that the spacing of the lines equals twice the width of the lines. Assume the same illumination as in (b). (3 marks)

(d) Design a practical monochromator to resolve spectral lines at 900.0 and 900.2 nm in first order. Follow the steps indicated.

(i) Sketch the system. Indicate the distance between the elements. Assume that the source is a point source. (3 marks)

(ii) State the purpose of each element. (3 marks)

(iii) Consider only the grating. How many lines must be illuminated to just resolve the spectral lines? State the criterion you are using to specify resolution. (3 marks)

(iv) Define *f*-number. Decide upon *f*-numbers for the optical elements in your design. Why would smaller or larger *f*-numbers be unacceptable? (4 marks)

(v) Specify a value for *a*/*b* for the grating, where a is the spacing between the lines of the grating and b is the width of the reflective part of each line of the grating. Why would larger or smaller values of *a*/*b* be unacceptable? (3 marks)

4. (a) Design a practical spatial filter/beam expander for a HeNe laser. For simplicity, assume that the output of a **perfect** HeNe laser can be considered to be uniform over the 1 mm aperture of the laser. Expand the beam to 1 cm in diameter and filter out any bright and dark areas on the laser beam. Explain your approach, show your calculations, and sketch a proper engineering diagram of your design. (15 marks)

(b) Now assume that the output of a perfect HeNe laser is a Gaussian beam with a full width half maximum of the irradiance of 1 mm. Redesign the spatial filter/beam expander to expand the beam 10× and to filter the beam as before. Discuss the changes that are necessary to accommodate the Gaussian beam and show your calculations. (10 marks)

5. The Fresnel equations for a dielectric medium can be written as

$$r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad t_{\perp} = \frac{2\sin(\theta_t)\cos(\theta_i)}{\sin(\theta_i + \theta_t)}$$

$$r_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \qquad t_{\parallel} = \frac{2\sin(\theta_t)\cos(\theta_i)}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

(a) Draw a diagram to define clearly the symbols and the *E* and *B* fields for the incident, reflected, and transmitted waves that are implicitly assumed in the Fresnel equations given above. What is the significance of the negative sign in the definition of $r_{\perp}$? (10 marks)

(b) Sketch maps of the reflection and transmission coefficients for internal and external reflection. Label the salient features of the sketches. (8 marks).

(c) Design a stacked plate polarizer to achieve maximum transmission of power, yet a polarization ratio of > 100:1. (7 marks)

6. (i) State the common lens types and the best form application fo each lens type. (4 marks)

(ii) Using ray tracing, find the image of an object placed 2 focal lengths in front of a perfect thin lens. (3 marks)

(iii) Now allow the lens to suffer from spherical aberration.

Trace rays through the system of (ii) to show the effect of spherical aberration. (2 marks)

(iv) Now allow the lens to suffer from chromatic aberration only. Trace rays through the system of (ii) to show the effect of chromatic aberration. (2 marks)

(v) Now allow the lens to be a thick lens. Trace rays through the system of (ii) to show the effect of the thickness. Ignore any other aberrations for this tracing. (2 marks)

(vi) Derive the matrices that are needed to trace the rays of light through a simple optical system that is composed of an object, a thick lens, and an image plane. Be sure to draw a labelled diagram to define your quantities. (10 marks)

(vii) Write the matrix equation that is required to find the image in (ii). Assume a thick lens. (2 marks)

Appendix

The intensity in the far-field as a function of the angle θ from the normal of the diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o\left[\frac{\sin(\beta)}{\beta}\right]^2\left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$

$$\beta = \frac{kb}{2}(\sin\theta_i + \sin\theta)$$

$$\alpha = \frac{ka}{2}(\sin\theta_i + \sin\theta)$$

double angle formulae:

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

The resolution R and the dispersion for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

$$D = \frac{m}{a\cos\theta}$$

For a circular lens of diameter D and image distance s, the full width of the central diffraction maximum between zeros is $2.44\ s\lambda/D$.