National Exams December 2009
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. 20 marks

2. 20 marks

3. (a) 10 marks, (b) 10 marks

4. 20 marks

5. 20 marks

6. 20 marks

7. (a) 10 marks, (b) 10 marks

8. 20 marks

1. Solve the initial value problem

$$y'' - 4y = 3t + e^{2t}, \qquad y(0) = 0, \, y'(0) = 2.$$

   Note that $'$ denotes differentiation with respect to t.

2. An elastic membrate in the $x_1 x_2$-plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched so that a point $P : (x_1, x_2)$ goes over into the point $Q : (y_1, y_2)$ given by

$$y_1 = 5x_1 + 3x_2,$$
$$y_2 = 3x_1 + 5x_2.$$

   Find the principal directions of the transformation. These are the directions of the position vectors $\mathbf{x}$ of all points P for which the direction of the position vector $\mathbf{y}$ of Q is the same or exactly opposite. What shape does the boundary circle take under the deformation?

3. Let P be the plane passing through the three points (0,1,2), (1,3,-1) and (2,0,1).

   (a) Find an equation representing the plane P.
   (b) Find the line of intersection between the plane P and the plane $x - 2y + z = 3$.

4. Find the general solution, $y(x)$, of the differential equation

$$2x^2 y'' + xy' - 3y = \frac{4}{x}.$$

   Note that $'$ denotes differentiation with respect to x.

5. Find the minimum value of the function $F(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $x + y - z + 1 = 0$

6. Evaluate the line integral of $\mathbf{F}$ over the curve C, where C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = 2 + x - 2y$, travelled clockwise as viewed from the positive z-axis, and let $\mathbf{F}$ be the vector function $\mathbf{F}(x, y, z) = 4z\mathbf{i} - y\mathbf{j} + y\mathbf{k}$.

7. Let $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

   (a) Find the linear approximation to f at (2,3,6).
   (b) Use this to approximate $\sqrt{(1.97)^2 + (3.02)^2 + (5.98)^2}$

8. Find the volume of the region bounded by the paraboloid $z = \frac{7}{4} + \frac{1}{4}(x^2 + y^2)$ and the plane $z = 4$ that lies outside the cone $z^2 - 4x^2 - 4y^2 = 0$.