National Exams December 2009

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

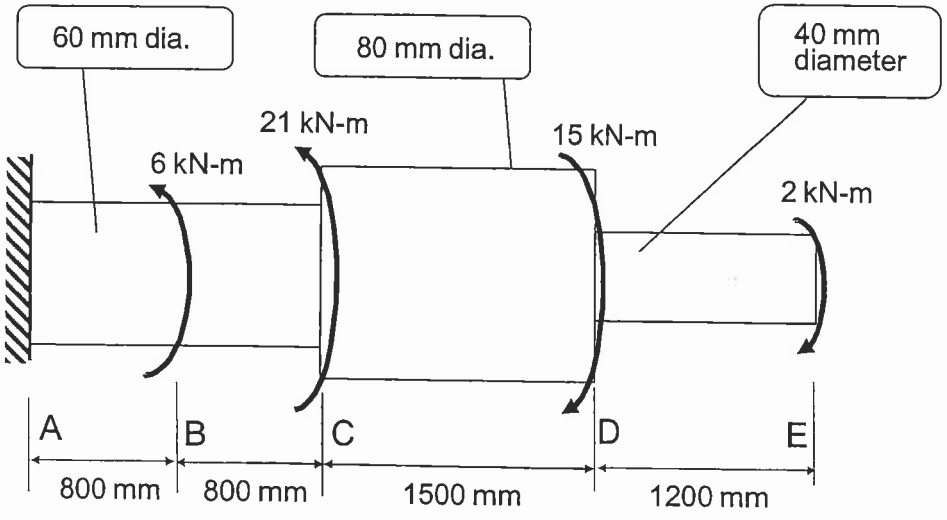
NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: A circular stepped shaft is subjected to four torques acting at B, C, D and E as shown. The entire shaft is made of steel with G = 80 GPa and a yield stress of 250 MPa. Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

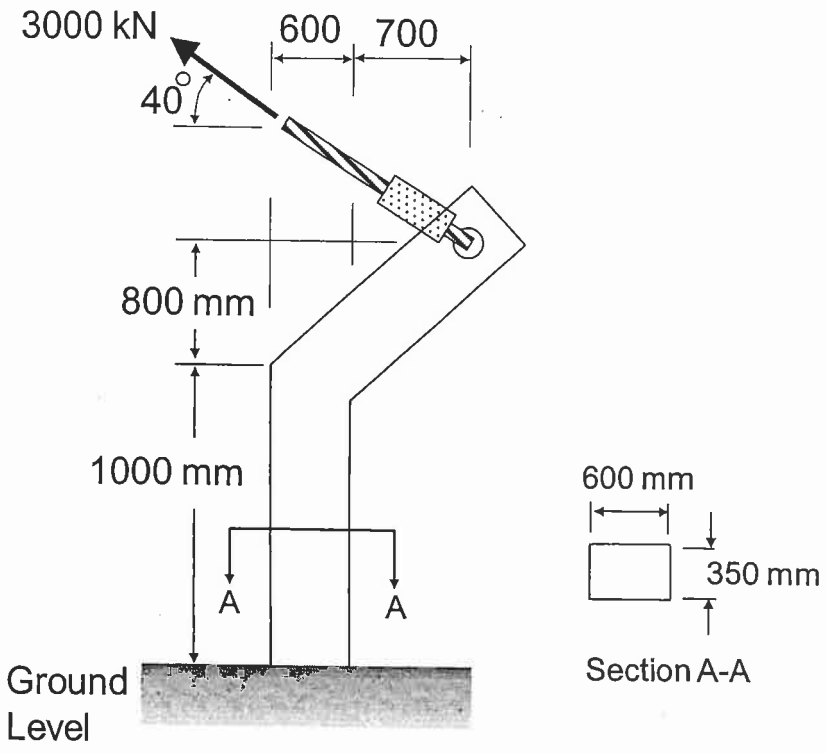
[12 marks] (a) determine the maximum shear stress in the stepped shaft and sketch the corresponding variation of shear stress along the shaft radius at this point.

[8 marks] (b) find the angle of twist at the end of the shaft (point E) and give your answer in degrees.



Question 2: A 3000 kN load is applied to a bent element with the cross section as shown below. The element is made of steel with a yield stress of 350 MPa and elastic modulus of 200 GPa.

[20 marks] Compute the normal stress distribution and shear stress distribution of the section at ground level (where the section is fixed). Show your answers on a sketch?

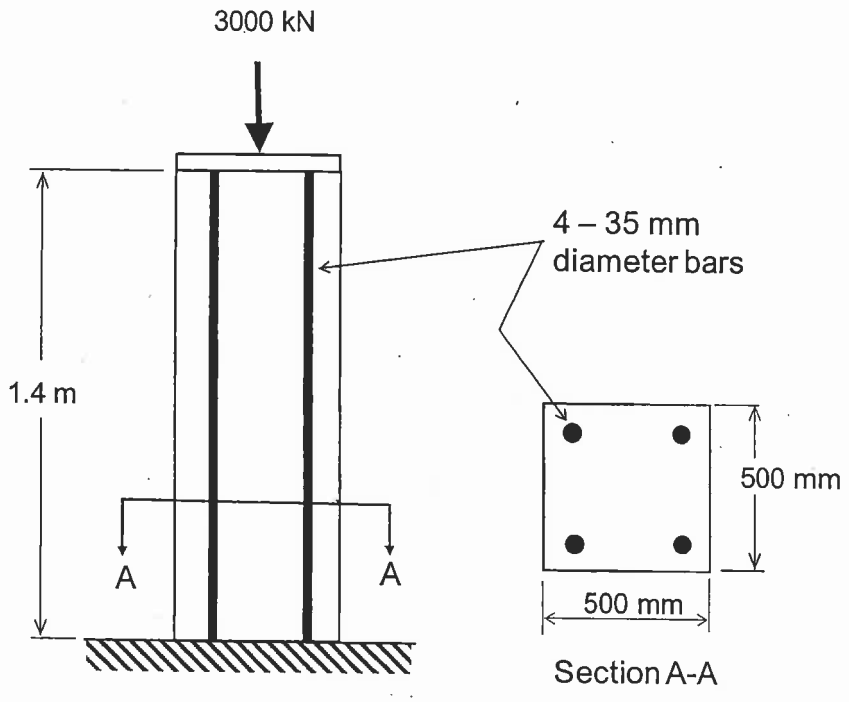


Question 3: A short 1.4 m high reinforced concrete post carries a compressive load of P = 3000 kN as shown. The column is 500 mm x 500 mm in cross-section and is reinforced with four 35 mm diameter steel bars. Assume the reinforcing steel has an elastic modulus of 200 GPa and yield strength of 400 MPa, while the concrete has an elastic modulus of 25 GPa and maximum compressive strength of 30 MPa.

[12 marks] (a) determine the forces carried by the concrete and reinforcing steel

[4 marks] (b) compute the corresponding shortening of the column

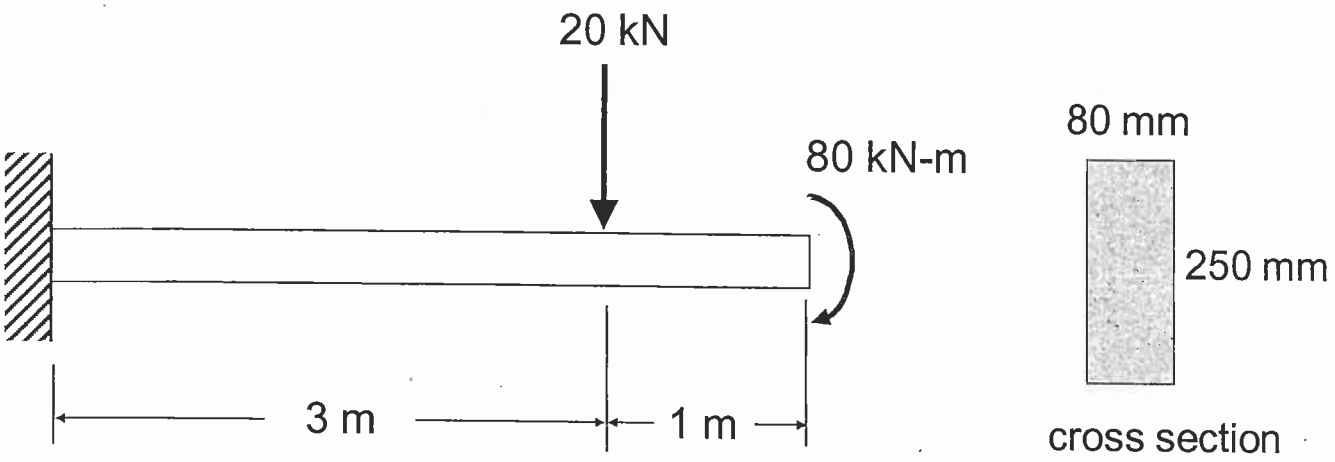
[4 marks] (c) what is the maximum load that can be carried by the column



Question 4: A cantilevered beam (fixed at the base) supports a concentrated load of 20 kN located 3 m from the fixed support and an 80 kN-m couple acting at the free end of the beam as shown. The beam is rectangular in cross-section with an 80 mm width and 250 mm depth. The beam is made from steel having an allowable normal stress of 260 MPa and elastic modulus of 200 GPa.

[18 marks] (a) determine the deflection and slope at the free end of the beam using the method of integration.

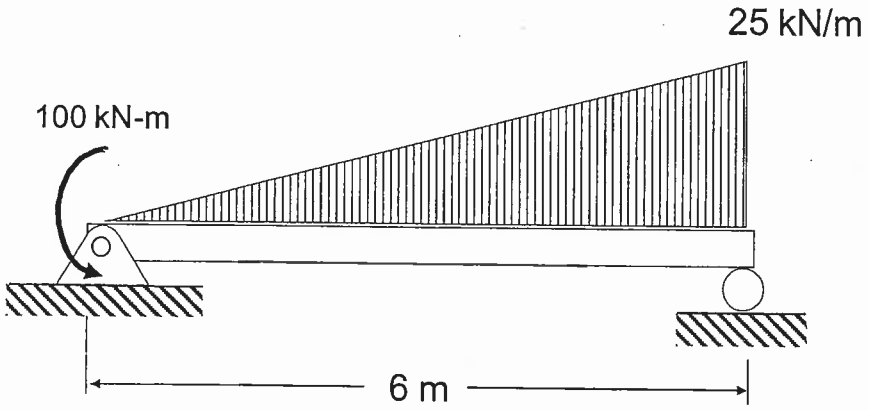
[2 marks] (b) recognizing that computation of deflection is a lengthy process using the method of integration, explain how you would compute the deflection of this beam in a design situation.



Question 5: A simply supported steel beam supports the loads shown below. The elastic modulus of the steel is 200 GPa.

[12 marks] (a) Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam.

[8 marks] (b) Next construct the shear force and bending moment diagrams. Remember to label points of maximum positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

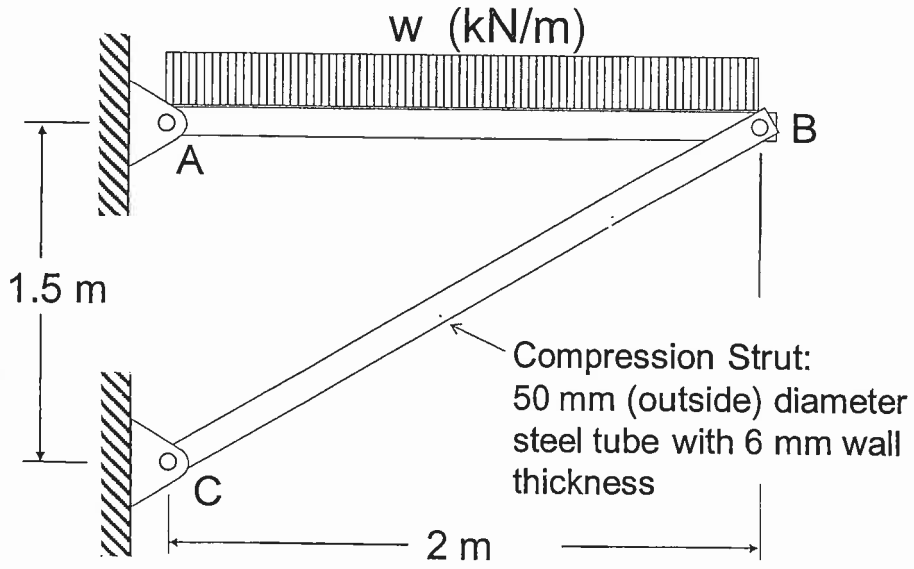


Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 6: For the structure shown below, the horizontal member supports a uniformly distributed load w in kN/m. The horizontal member is propped up by an inclined strut fabricated from a steel pipe having an outside diameter of 50 mm and wall thickness of 6 mm. Assume the compressive strut is pinned at both ends with in-plane buckling only. The steel used for the strut has an elastic modulus of 200 GPa and yield strength of 350 MPa.

[20 marks] Determine the largest load w that can be supported by the structure without causing the strut to buckle. Consider in-plane buckling only and use a factor of safety of 2 for the Euler buckling load. The safety factor for yielding of the steel is 1.5.

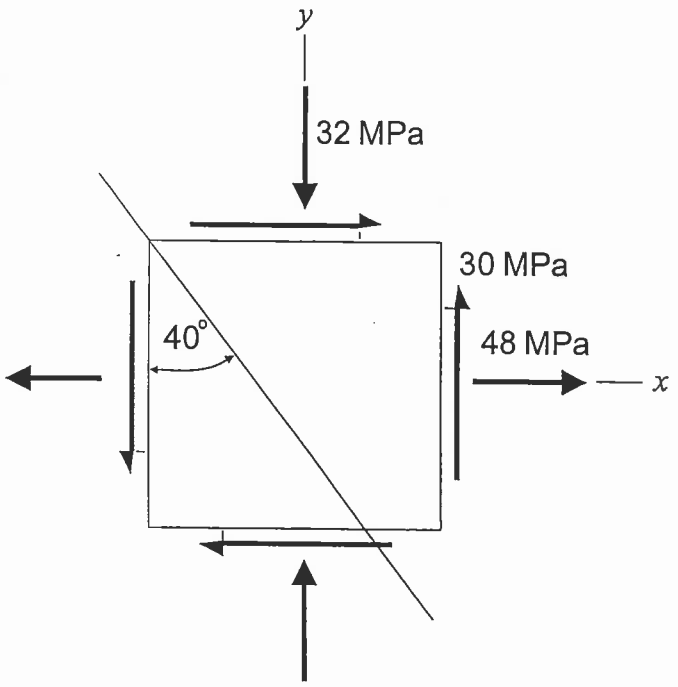


Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$

Question 7: Use the Mohr's circle solution (*not* the transformation equations) to determine the following: [20 marks]

(a) stresses acting on an inclined plane orientated 40 degrees to the vertical as shown. Show your answer on a properly orientated element.

(b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.

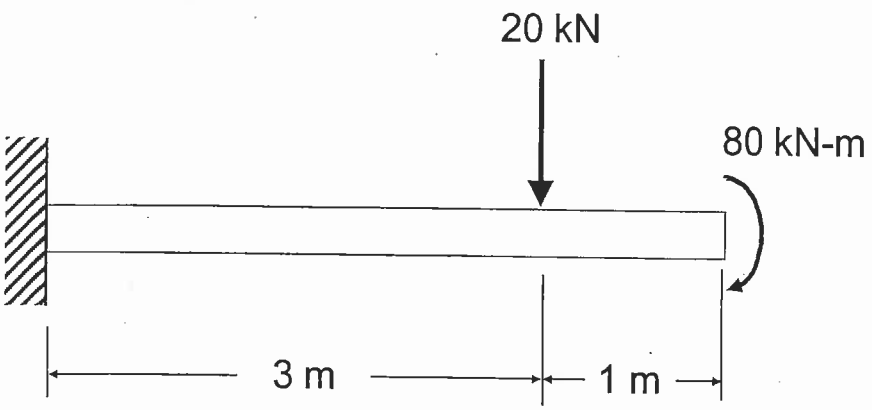


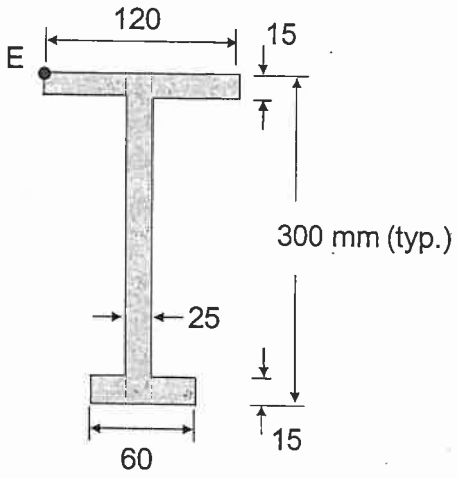
Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 8: A cantilevered beam (fixed at the base) supports a concentrated load of 20 kN acting 3 m from the fixed support and an 80 kN-m couple acting at the free end of the beam as shown. The beam has the cross-section shown below and is made from steel having an allowable normal stress of 260 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel used equals 200 GPa.

[18 marks] (a) Determine whether the beam is safe.

[2 marks] (b) Determine the shear stress at point E on the cross section at the base of the cantilever.





beam cross-section
(all dimensions in mm)