NATIONAL EXAMS December 2009
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two approved calculators, a Casio or Sharp Models.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

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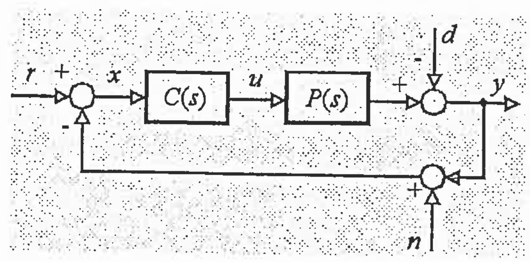
1. Consider the feedback system below with, $C(s)=\dfrac{sK_p+K_i}{s}$, $P(s)=\dfrac{-s+4}{(s+4)(s^2+s+1)}$

(a) With $K_p = 0$, determine the range of K_i for closed loop stability.

(b) With $K_p = 0$, determine the values of K_i for which the controller yields a gain margin of at least 6db and a phase margin of at least 50 degrees.

(c) Determine the steady state value of $x(t)$ as a function of K_p and K_i when $r(t)$ = a ramp with slope 2, and $d(t) = 2$, and $n(t) = 0$.



2. Consider the model,

$$\dot{x}(t)=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}x(t)$$
$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}x(t)$$

(a) Show that the model is such that $y(t)$ is a sinusoid whose phase depends on $x(0)$.

(b) Design an observer for $x(t)$ such that the observer poles are at s = -10, -10.

(c) Determine the transfer function, $\dfrac{\hat{y}(s)}{y(s)}$, where $\hat{y}(s)$ denotes the estimate of $y(s)$.

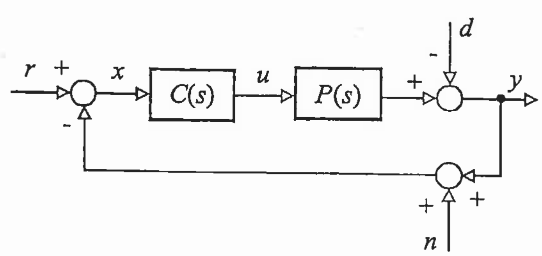
3. Consider the multi-input-multi-output feedback system below with,

$$P(s)=\begin{bmatrix}\dfrac{1}{s} & \dfrac{-1}{(2s+2)}\\ 1 & \dfrac{4}{s(s+1)}\end{bmatrix},\quad C(s)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

(a) Determine the transfer function, $S(s)$, that relates $y(s)$ to $d(s)$.

(b) Justify whether $S(s)$ is stable or not.

(c) Find a state space model for $P(s)$.



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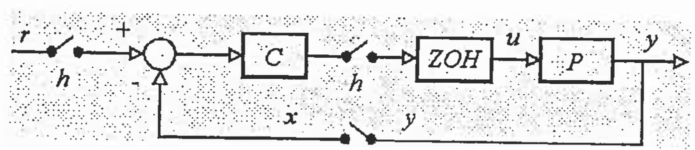
4. Measurements of the frequency response for an unknown but stable first order system are recorded as follows,

Frequency	Gain	Phase Shift
0 rad/s	9.543 db	0
1 rad/s	5.563 db	-108.4 deg
2 rad/s	4.228 db	-139.4 deg

(a) Determine the transfer function, $P(s)$. *Note*: the transfer function of the system may have both numerator and denominator terms.

(b) Draw the associated unit step response being careful to identify the key features.

5. Consider the sampled data system shown on the right. The input to the ZOH, the set-point, r, and the output, y, are uniformly sampled with a sample period of $h = 0.1$ with $C(s)$ and $P(s)$ given by,



$$C(s) = K, \quad P(s) = \frac{e^{-s/10}}{s(s+1)}$$

(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.

(b) Sketch and annotate the root locus as K varies from zero to infinity.

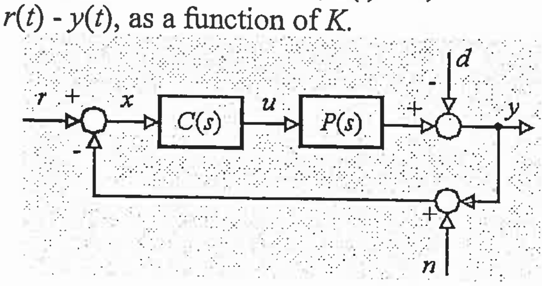
(c) Determine the limiting value of K for stability.

6. Consider the (continuous time) feedback system below with, $C(s) = K, \quad P(s) = \dfrac{e^{-s/10}}{s(s+1)}$.

(a) Determine the range of K that results in closed loop stability.

(b) Determine the phase margin when $K = 1$ and sketch the associated Nyquist plot.

(c) The system is stable and operating with a sensor bias, $n(t) = 0.3$, disturbance, $d(t) = 0$, and set-point, $r(t) = 1$. Determine the tracking error, $e(t) = r(t) - y(t)$, as a function of K.



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Inverse Laplace Transforms	
$F(s)$	**$f(t)$**
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^n e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\dfrac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\dfrac{2t^n e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	**$f(nT)$**
$\dfrac{Kz}{z-a}$	Ka^n
$\dfrac{(C+jD)z}{z-re^{j\varphi}}+\dfrac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^n\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{(z-a)^r}$, $r=2,3\ldots$	$\dfrac{Kn(n-1)\ldots(n-r+2)}{(r-1)!a^{r-1}}a^n$

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Table of Laplace and z-Transforms
(h denotes the sample period)

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$