National Exams December 2009

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Question 1:

For the system of Figure 1, determine the range of K for which the system is stable.

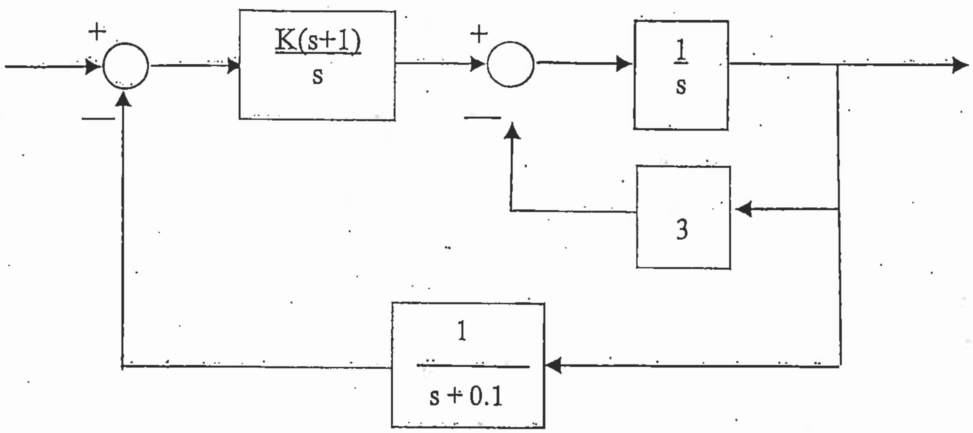

Figure 1

Question 2:

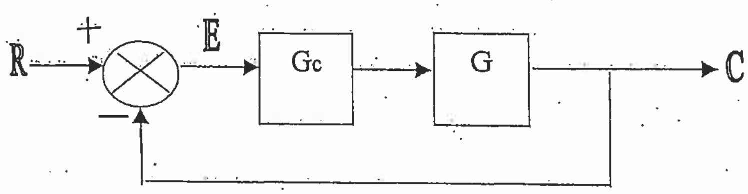

Figure 2

a) Figure 2 models a pressure control system with a plant $G = 1/[(s+1)(s+3)]$ consisting of two simple lags, and PI control $Gc = K(s+2)/s$:

i) Sketch the loci of the closed-loop system poles for varying K.

ii) Find, reasonably accurately, the value of K for a damping ratio 0.5 for the dominating pair of poles.

b) Plot the root loci for a system with the loop gain function (i.e. open loop transfer function)

$$\frac{K}{(s+2)(s+6)(s^2+8s+20)}$$

and find the limiting value of K for stability.

Question 3:

Determine the maximum value for the Bode gain K_B which will result in a gain margin of 6 db or more and a phase margin of 45° or more for the system with the open-loop frequency response function

$$GH(j\omega) = \frac{K_B}{j\omega(1+j\omega/5)^2}$$

Question 4:

a) Using the Laplace transform technique, find the transient and steady-state responses of the system described by the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 1$ with initial conditions $y(0^+)$ and $\left.\frac{dy}{dt}\right|_{t=0+} = 1$.

b) Using the Laplace transform technique, find the unit impulse response of the system described by the differential equation $\frac{d^3y}{dt^3} + \frac{dy}{dt} = x$.

Question 5:

a) Determine if the following characteristic equation represents a stable system:

$s3 + 4s^2 + 8s + 12 = 0$

b) The characteristic equation of a given system is:

$s^4 + 6s^3 + 11s^2 + 6s + K = 0$

What restrictions must be placed upon the parameter K in order to insure that the system is stable?

c) The block diagram below depicts a first order lag system.

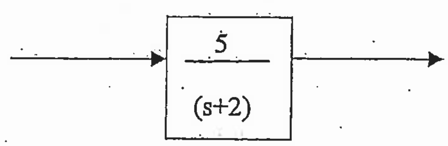

Figure 3

Derive an expression for the output if the input, (at time t = 0), is a unit ramp. How would you obtain the response to a unit impulse from the expression which you have derived?

Question 6

For the stable system

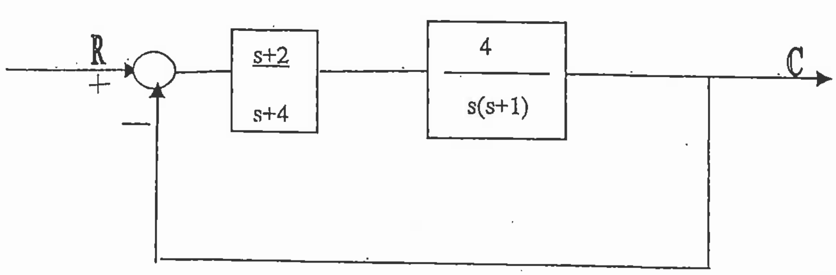

Figure 4

a) Determine the system type.

b) Find the steady-state error for a unit step input, a unit ramp input, and a unit parabolic input.

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n^2\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$

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Semi-Logarithmic
3 Cycles x 10 to the inch