National Exams May 2009
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 6 marks, (b) 14 marks
2. 20 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Consider the quadratic form $5x^2 + 24xy - 5y^2 = 13$.
   (a) What type of conic section is represented by the above quadratic form?
   (b) Transform the quadratic form to principal axes.

2. Find the general solution to the differential equation
$$y'' + 2y' - 3y = 9x^2 + e^{-3x}$$
Note that $'$ denotes differentiation with respect to x.

3. Find the volume of the solid whose base is the region bounded by the parabola $y = 3x - x^2$ and the line $y = 3 - x$ and whose cross-sections perpendicular to the x-axis are semicircles with diameters on the x-y plane.

4. Find the line tangent to the intersection of the surfaces
$$6x^2 + y^2 - z = 4$$
and
$$4x^2 + y^2 + z^2 - 4y - 4z + 8 = 0$$
at the point $(1, 0, 2)$.

5. Let S be the boundary of the region enclosed by the paraboloid $z = x^2 + y^2 - 2$ and the plane $z = 2$ and let
$$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + 2xyz\mathbf{j} - xz^2\mathbf{k}.$$
Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{n}$ is the unit outward normal on S.

6. Find the general solution to the differential equation $x^2y' + 2xy = \cos^2 x$.

7. At what angle does the line represented parametrically by $x = 2 - t$, $y = t$, $z = 2 + 2t$ intersect the hyperboloid $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

8. Find the maximum and minimum values of $f(x, y, z) = x + y - z$ over the sphere $x^2 + y^2 + z^2 = 1$.