NATIONAL EXAMINATIONS MAY 2009

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumption made.
2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.
3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. (a) 15 marks ; (b) 5 marks
3. (A) 10 marks ; (B) 10 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. (A) 10 marks ; (B) 10 marks
8. 20 marks

1.Find the eigenvalues and eigenfunctions of the following Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0 \qquad ; y(0)=0 \qquad ; y'(L)=0$$

2. (a) Find the Fourier series expansion of the periodic function F(x) of period p=8.

$$F(x) = \begin{cases} 3 + x & -4 < x \leq 0 \\ 3 - x & 0 < x \leq 4 \end{cases}$$

(b) Use the Fourier series expansion you obtained in (a) to find an appropriate series that will enable you to evaluate $\pi^2/16$.

3. (A) Prove that the coefficients a and b of the least-squares line y = a + bx that fits the set of n points (x_i, y_i) are as follows:

$$b = \frac{n\sum_{i=1}^{i=n} x_i y_i - \sum_{i=1}^{i=n} x_i \sum_{i=1}^{i=n} y_i}{n\sum_{i=1}^{i=n} x_i^2 - (\sum_{i=1}^{i=n} x_i)^2} \qquad ; \qquad a = \frac{\sum_{i=1}^{i=n} y_i}{n} - b\frac{\sum_{i=1}^{i=n} x_i}{n}$$

3.(B) It is suggested that the following set of n=6 points (z_i, y_i) are related by an equation of the form $y = a + b/\sqrt{z}$. Use the results obtained in 3(A) to find the least-squares estimate of a and b. (Hint: Let $x=1/\sqrt{z}$).

z_i	1	2.25	4	9	16	25
y_i	24	16	14	11	10	8

4.Find the interval of convergence for the following series and test for convergence or divergence at the end points if the interval is finite.

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{3^n \sqrt{n}}$$

5. Use Romberg's algorithm with n = 2 to evaluate the area bounded by $f(x)=(7+x^3)^{4/3}$, x=1, x=3 and y=0.

Hint: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(0,0)		
R(1,0)	R(1,1)	
R(2,0)	R(2,1)	R(2,2)

where

$$R(0,0)=\frac{1}{2}(b-a)[f(a)+f(b)]$$

$$R(n,0)=\frac{1}{2}R(n-1,0)+h\sum_{k=1}^{2^{n-1}} f[a+(2k-1)h]$$

where $h=\dfrac{b-a}{2^n}$

$$R(n,m)=R(n,m-1)+\frac{1}{4^m-1}[R(n,m-1)-R(n-1,m-1)]$$

6. Consider the following linear simultaneous differential equations:

$$\frac{d^2x}{dt^2}+5x-2y=0$$

$$\frac{d^2y}{dt^2}-2x+2y=0$$

Find y(t) only if x(0) = −2 ; x'(0)=0; y (0) = 4; y'(0)=0.

7(A).(a) Consider the equation $e^x-5x-3=0$. Use the method of false position twice to find the root that lies between a = 2 and b = 3. (b) Apply Newton's method twice to get a better approximation to the root starting with the last approximation obtained in (a). (Note: Carry seven significant digits in your calculations).

7.(B) One root of the equation $x^3 - 12x^2 + 36x - 30 = 0$ is close to $x_0 = 1$. Use the following iterative formula twice to find a better approximation of this root. (Note: Carry seven significant digits in your calculations).

$$x_{i+1} = x_i - \frac{f(x_i)}{f^{(1)}(x_i) - \dfrac{f(x_i)f^{(2)}(x_i)}{2f^{(1)}(x_i)}}$$

Hint: Let $f(x) = x^3 - 12x^2 + 36x - 30 = 0$. Note that $f^{(1)}(x)$ and $f^{(2)}(x)$ denote the first and second derivative of f(x) respectively.

8.Solve the following linear system using Gaussian elimination with partial pivoting. (Note: No other method will be accepted).

$$\begin{aligned} 4X_1 + 3X_2 - 7X_3 &= 13 \\ 8X_1 - 5X_2 - 10X_3 &= 29 \\ -2X_1 + 2X_2 + X_3 &= -5 \end{aligned}$$