PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – May 2009

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) that he/she has had made with the answer.

2. The examination paper is open book and so candidates are permitted to make use of any textbooks, references or notes that they wish to use.

3. Any non-communicating calculator is permitted. A calculator that can handle small matrices will speed the solving of the problems. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.

4. Candidates are required to attempt five questions. Solve all problems using finite element method.

5. All questions carry the same value. Indicate which five questions are to be marked on the cover of the first examination workbook.

PROBLEM 1 (20 POINTS)

Consider the following differential equation:

$$\frac{d^2u}{dx^2} = 6x + 2 \qquad \text{for} \quad 0 \le x \le 1$$

subject to boundary conditions:

$$u(0) = 0 \quad ; \quad u(1) = 1$$

a. Comment on the boundary conditions.

b. Using the trial function:

$$\phi_j = x^j (1 - x)$$

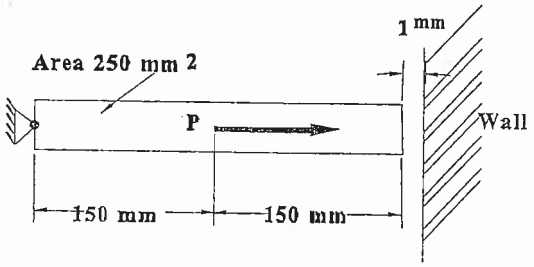
find a two parameter (N=2) Galerkin solution.

c. Compare the approximate solution found in (b) with the exact solution.

PROBLEM 2 (20 POINTS)

A load P = 75 kN is applied to the truss shown in the figure below.
A gap of 1.0 mm exists between the truss and the wall. Calculate the stresses and the reactions.

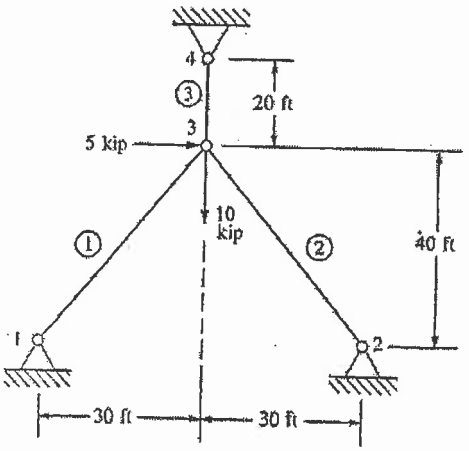
The area of the truss is 250 mm^2 and E = 20 x 10^3 N/mm^2



PROBLEM 3 (20 POINTS)

Determine the displacement components at node 3 and the element forces for the plane truss shown below.

A = 3 in^2 and E= 30 x 10^6 lb/in^2 for all elements.



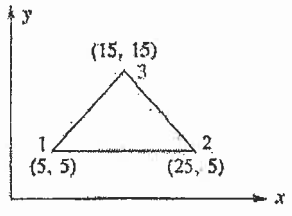
PROBLEM 4 (20 POINTS)

Consider the plain strain triangular element shown below. The nodal displacements are given as:

u_1= 0.003 mm	**u_2= 0.0 mm**	**u_3= 0.005 mm**
v_1= 0.002 mm	**v_2= 0.0 mm**	**v_3= 0.0 mm**

Determine the element stresses σ_x, σ_y, τ_{xy} and σ_1 and σ_2 and the principal angle θ_p

E= 70 GPa and use unit thickness for plane strain. All coordinates are in millimetres.

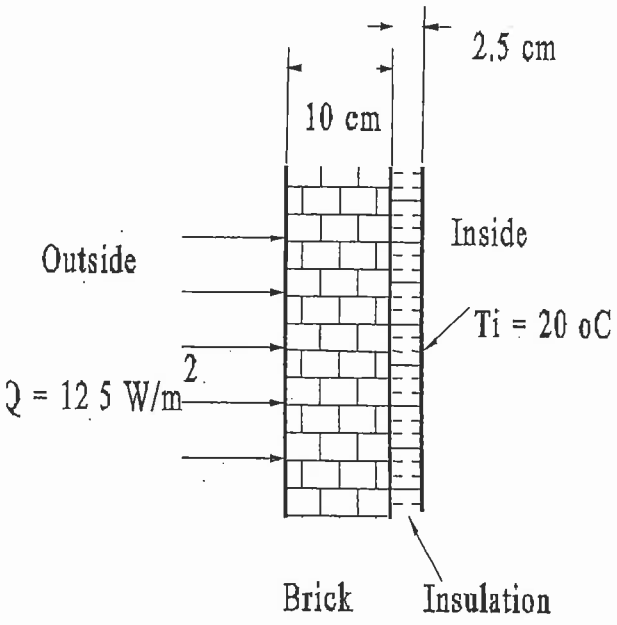


PROBLEM 5 (20 POINTS)

The exterior surface of a wall is heated by sun at a rate of $q = 125\ W/m^2$.
The wall is composed of a layer of brick and a layer of thermal insulation. The brick is 10 cm thick, and has a conductivity of $k_{brick} = 0.75\ W/m^oC$. The insulation is 2.5 cm thick and has a conductivity of $k_{insulation} = 0.04\ W/m\ ^oC$.
The inside surface of the wall is maintained at $T_i = 20^oC$.

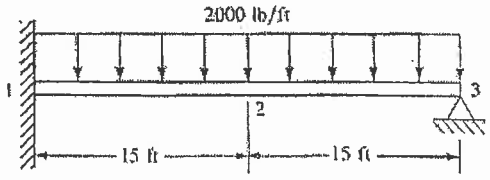
Assuming that all sun heat penetrates the wall (no heat losses from the outside surface of the wall), calculate the two elements solution for the temperature distribution in the wall.

Formulate the problem, using finite elements analysis. Assume that there is no internal heat generation, and that the heat flow is unidimensional.



PROBLEM 6 (20 POINTS)

For the beam under the loading shown below determine the nodal displacements and slopes and the reactions.
$I = 200\ in^4$. $E = 30 \times 10^6\ lb/in^2$



PROBLEM 7 (20 POINTS)

PART A. (10 points.)

Explain in a sentence or two the following concepts:

- skyline solution
- symmetric banded matrix
- Gauss elimination
- to what does the term *degree of freedom* refer?

PART B.(10 points)

Number the node and the elements for the truss shown below. Make use of the symmetry in such a way as to reduce the model. Draw the new model and state clearly all your assumptions. Do not make any calculations. Same cross section of each member.