National Exams May 2009

98-Phys-A7, Optics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.

2. This is a **CLOSED BOOK EXAM**.

3. Candidates may use one of two calculators, the Casio or Sharp **approved models**.

4. Answers to question 1 plus any **three** of questions 2 to 6 constitutes a complete exam paper.

5. Answer question 1 in the space provided on the exam paper.

6. The first three questions as they appear in the answer book will be marked.

7. Each question is of equal value. Question 1 is mandatory.

1. Answer question one in the space provided. A phrase, or a diagram and a phrase, is all that is required in most cases. [25 marks, one mark for each letter]

a) This exam is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) State the *law of reflection*

g) State the *law of refraction.*

h) Complete any two rows of the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

Maxwell's equations are:

$$\nabla \cdot D = \rho_f$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J_f + \frac{\partial D}{\partial t}$$

i) Define and give SI units for the *E* that appears in Maxwell's equations.

j) Define and give SI units for the *B* that appears in Maxwell's equations.

k) Define D in terms of E. Assume a linear, homogeneous, isotropic dielectric.

l) Define H in terms of B. Assume a linear, nonferromagnetic, homogeneous, isotropic medium

m) Over what frequency range are Maxwell's equations valid?

n) Maxwell's equations can be solved under certain approximations to find that E and B are transverse plane waves. What are the approximations?

o) Describe a transverse plane wave.

p) For most problems an analysis in terms of monochromatic plane waves is not restrictive. Why?

q) Write an equation for the *Poynting vector* and give the SI units for the quantity the Poynting vector represents.

r) What is the *refractive index of a material* defined to be equal to?

s) What defines the *direction of polarization* of light?

t) What is the difference between *interference* and *diffraction*?

u) What does the concept of *spatial coherence* deal with?

v) What does the concept of *temporal coherence* deal with?

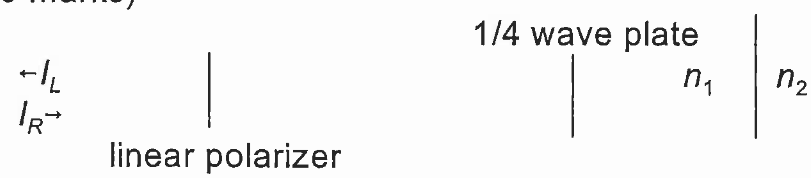
w) Define *dispersion* as it relates to the refractive index.

x) At what speed does light appear to propagate?

y) At what speed does the energy of a beam of light propagate?

2. (a) The energy density of light of wavelength = 660 nm is reduced to one quarter of its original value by propagation through 342 cm of sea water. What is the absorption coefficient for light of this wavelength? At what depth is light of wavelength = 660 nm reduced to 1% of its original value? (4 marks)

(b) Determine I_L / I_R for the following system, assuming that the 1/4 wave plate and linear polarizer are perfectly antireflection coated. Take I_R to be a plane, unpolarized beam propagating along a direction that is normal to the surface of all three elements. Take the axis of the linear polarizer to be oriented at 45 deg to the principal axis of the 1/4 wave plate and take $n_2 > n_1$. (8 marks)



(c) Show that a ½ wave plate rotates the plane of polarization by an angle of 2θ where θ is the angle between the input polarization and one principal axis of the ½ wave plate. (8 marks)

(d) Deduce the nature of light that is consistent with the two analyses described below. Assume perfect optical elements and alignments. Explain your reasoning (5 marks).

(i) When a polarizer is rotated in the path of the light, there is some intensity variation but no position of the polarizer gives zero intensity. (ii) The polarizer is set for maximum transmission. A 1/4 wave plate is placed in front of the polarizer and the 1/4 wave plate is aligned with a principal axis parallel to the transmission axis of the polarizer. Rotation of the polarizer can now produce zero intensity.

3. Answer the following questions for the monochromator described below. Note that the optical axes of the input slit, lens #1, transmission grating, and lens #2 lie on the same line. The output slit can be translated in a direction perpendicular to the optical axes of the other elements.

input slit	lens #1	transmission grating	lens #2	output slit

distances: input slit to lens #1:50 cm; lens #1 to grating: 30 cm; grating to lens #2: 30 cm; lens #2 to output slit: 50 cm.

Assume that both lenses are 10 cm in diameter and that the grating is square and larger than the lenses.

(a) Explain the function of (5 marks):
(i) the input slit
(ii) the output slit
(iii) the grating
(iv) lens #2
(v) lens #1

What is the focal length of lens #2?(1 mark)

(b) Calculate the resolution in first order and for a wavelength of 1000 nm. Assume that the grating has 500 grooves/mm, that the output slit is 20 μm wide, that lens #2 is diffraction limited and forms the aperture stop, and that the grating is illuminated with a plane wave at normal incidence. (5 marks)

(c) Now estimate the resolution of the grating only at 1000 nm and in first order. Use Rayleigh's criterion and assume that the illuminated area of the grating is 10 cm wide and that the grating has 500 grooves/mm. Draw a sketch that defines Rayleigh's criterion, set-up the equations, and solve for the resolution. (5 marks)

(d) Estimate the reduction in resolution that would occur if the input was an extended source 0.5 mm wide. Use the same parameters as in part (b). (5 marks)

(ii) The efficiency of the grating (i.e., percentage of light diffracted into orders) was measured for TE and TM polarized light. The grating efficiency for light polarized in the TE direction was greater than for light polarized in the TM direction. How would you explain this effect? What determines the TE and TM designations of the light? (4 marks)

4. Solve for the intensity reflection and transmission coefficients at normal incidence for a planar interface. Follow the steps indicated.

(a) Draw a sketch of the problem and state the approach used to solve the problem. Indicate the directions of the *E* and *B* fields for the incident, reflected, and transmitted fields. Draw the coordinate system used. (5 marks)

(b) State the approximations inherent in the solution and method of solution. (2 marks)

(c) Solve for the amplitude reflection and transmission coefficients. (4 marks)

(d) Provide definitions for the intensity reflection and transmission coefficients, and then evaluate the coefficients. (4 marks).

(e) Why is it necessary to solve for only one polarization in this problem?. (1 mark).

(f) If the frequency of the light in the incident medium is 10^{13} Hz, and the ratio of refractive indices is 2, what is the frequency of light in the transmitted medium? (1 mark)

(g) Sketch the intensity reflection and transmission coefficients as a function of the angle of incidence for (i) internal reflection and (ii) external reflection. Label all important features and remember that there are 2 orthogonal polarizations. (5 marks)

(h) Use the curves in part (g) to explain the conditions when polarizing sun glasses are most effective. (3 marks)

5. (a) Use ray tracing to find the image for a point source located 2 focal lengths in front of a perfect lens and on the optic axis of the lens. (3 marks)

Identify a marginal ray and show where this family of marginal rays will focus. Identify a paraxial ray and show where this family of paraxial rays will focus. (3 marks)

(b) Now allow the lens to suffer from chromatic aberration only. Trace rays through the system of (a) to show the effect of chromatic aberration. (2 marks)

(c) Now allow the lens to be a thick lens. Trace rays through the system of (a) to show the effect of the thickness. Ignore any other aberrations for this tracing. (2 marks)

(d) Derive the matrices that are needed to trace the rays of light through a simple optical system that is composed of an object, two thick lens, and an image plane. Be sure to draw a labelled diagram to define your quantities. (10 marks)

(e) A compound lens is composed of two thin lenses separated by 5 cm. The first lens has a focal length of 10 cm and the second lens has a focal length of −10 cm. What is the focal length of the system. Using ray tracing, find the image for an object placed 20 cm in front of the first lens.(5 marks)

6. (a) Draw a cross section through a piece of single mode optical fibre. Label the features in the cross section. Explain how single mode optical fibre works. Give typical dimensions for the features in the cross section and explain why smaller or larger features would be inappropriate. (10 marks).

(b) The propagation dispersion per km of optical fibre can be approximated as

$$\frac{d\tau}{d\lambda} = 0.085\,\lambda\left(1 - \frac{\lambda_0^{\,4}}{\lambda^4}\right)\frac{ps}{nm \times km}$$

where τ is the propagation time for light of wavelength λ, and λ_0 is the wavelength of zero dispersion.

(i) If a diode laser under modulation has a rectangular linewidth of 5 nm full width, find the range of centre wavelengths for which the diode laser can be used in a system with 30 km of fibre and modulation at 500 Mbit/s. Follow the steps indicated.

(ii) Plot the power spectral output of the laser, i.e., the output as a function of wavelength. Assume a total power of *P* mW. Label the mean wavelength of the laser, the maximum and minimum wavelengths, and scale the ordinate. (3 marks)

(iii) Sketch the power output of the laser as a function of time for three periods of the modulation and before any dispersion is introduced . Assume on-off signalling. Label times on the sketch. (3 marks).

(iv) Sketch the output of the fibre after propagation over a length of fibre to show the effects of the dispersion. Indicate with additional sketches the criteria you will use to find the range of centre wavelengths that can be used. (6 marks)

(v) Solve for the range of centre wavelengths. (3 marks)

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Appendix

The intensity in the far-field as a function of the angle θ from the normal of the diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o \left[\frac{\sin(\beta)}{\beta}\right]^2 \left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$

$$\beta = \frac{kb}{2}(\sin\theta_i + \sin\theta)$$

$$\alpha = \frac{ka}{2}(\sin\theta_i + \sin\theta)$$

double angle formulae:

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

The resolution R and the dispersion for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

$$D = \frac{m}{a\cos\theta}$$

For a circular lens of diameter D and image distance s, the full width of the central diffraction maximum between zeros is $2.44\ s\lambda/D$.