National Exams December 2010

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

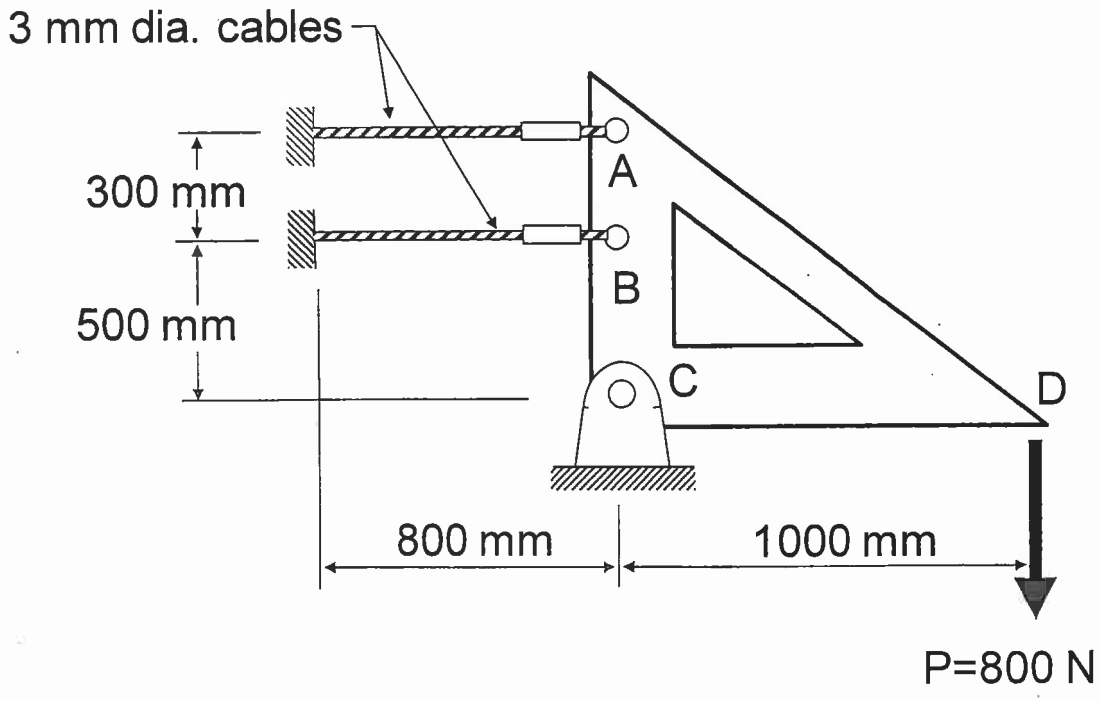
Question 1: A rigid triangular frame is pivoted at C with a 16 mm diameter pin and supported by two 3 mm diameter cables at A and B as shown below. The cables are made of steel with an elastic modulus E of 200 GPa and yield stress of 240 MPa. A vertical load of P = 800 N is applied to the triangular frame at D.

Determine the following:

[12 marks] (a) forces carried by the two steel cables

[4 marks] (b) corresponding displacement at point D where the load P is applied

[4 marks] (c) shear stress in the 16 mm diameter pin at C, assuming the pin is loaded in double shear.

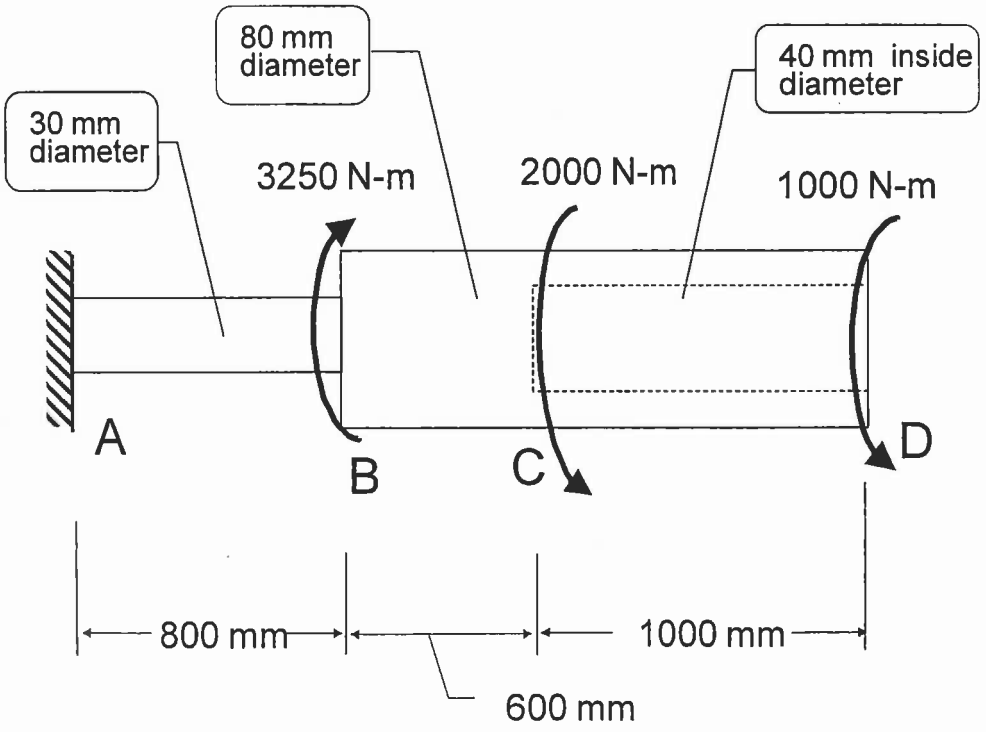


Question 2: A circular stepped shaft ABCD is subjected to the torques shown. Part of the shaft CD is hollow. The shaft is made of steel with an elastic modulus E = 200 GPa, shear modulus G = 80 GPA, and yield stress (in shear) τ_y = 210 MPa.

[12 marks] (a) determine the maximum shear stress in the stepped shaft, and sketch the corresponding variation of shear stress along the shaft radius at this point.

[6 marks] (b) find the angle of twist at the end of the shaft (point D) and give your answer in degrees.

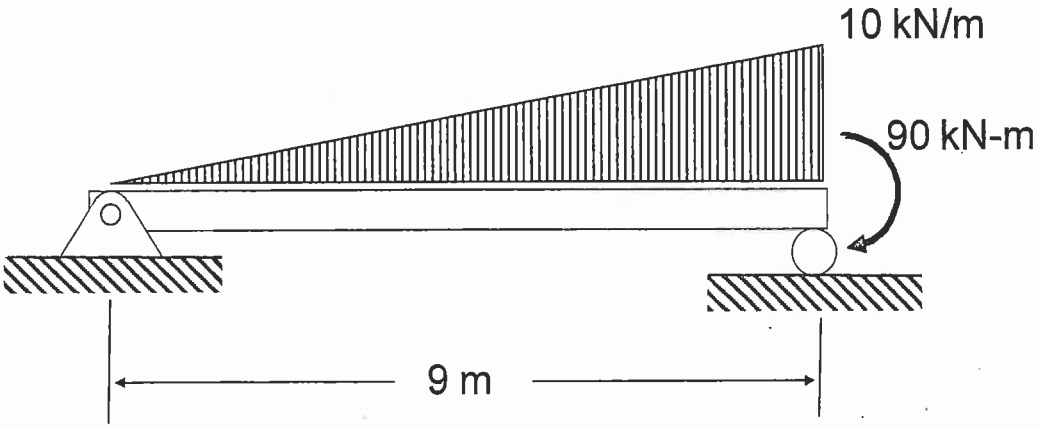
[2 marks] (c) what would happen if the loads on the shaft were doubled?



Question 3: A simply supported beam is subjected to a triangular distributed load and an applied clockwise couple (at the right support) as shown.

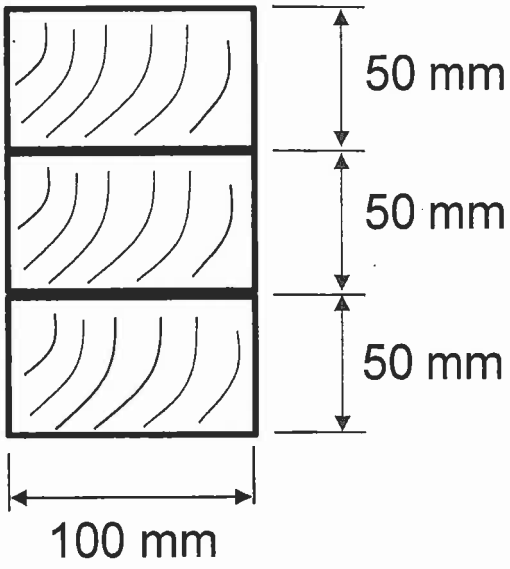
[20 marks] Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam. Next construct the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).



Note that **no credit** will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 4: [20 marks] A cantilever beam has a built up section by gluing together three 100 mm x 50 mm boards (actual dimension) to form a solid beam 100 mm wide x 150 mm high in cross section. The allowable shear stress in the glued joints is 350 kPa, while the wood has an allowable bending (normal) stress of 11 MPa and allowable shear stress of 0.5 MPa. Given that the beam is a 1.5 m long cantilever, what is the allowable concentrated load P that can be applied at the free end of the beam? (Disregard the weight of the beam).

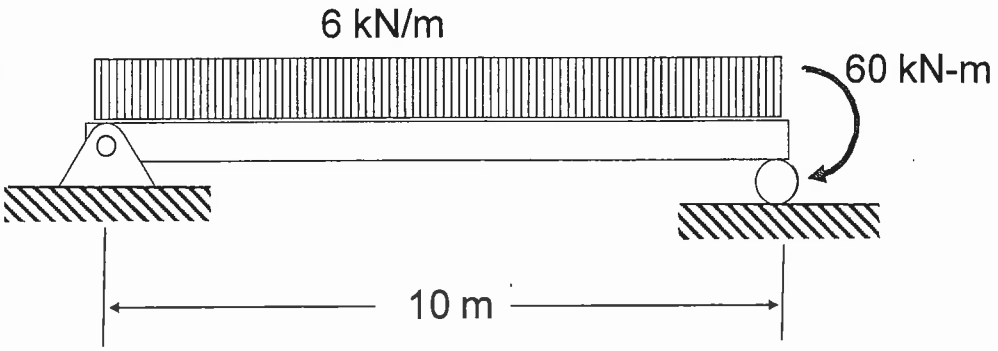


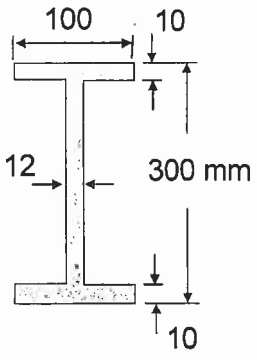
Beam Cross-Section

Question 5: A simply supported wide flange beam is subjected to a uniformly distributed load and applied clockwise couple (at the right support) as shown. The beam section show below is made with steel having an allowable normal stress of 240 MPa and elastic modulus of 200 GPa.

[18 marks] (a) Determine the maximum deflection in the beam using the method of integration.

[2 marks] (b) How would you compute the deflection of this beam in a design situation?





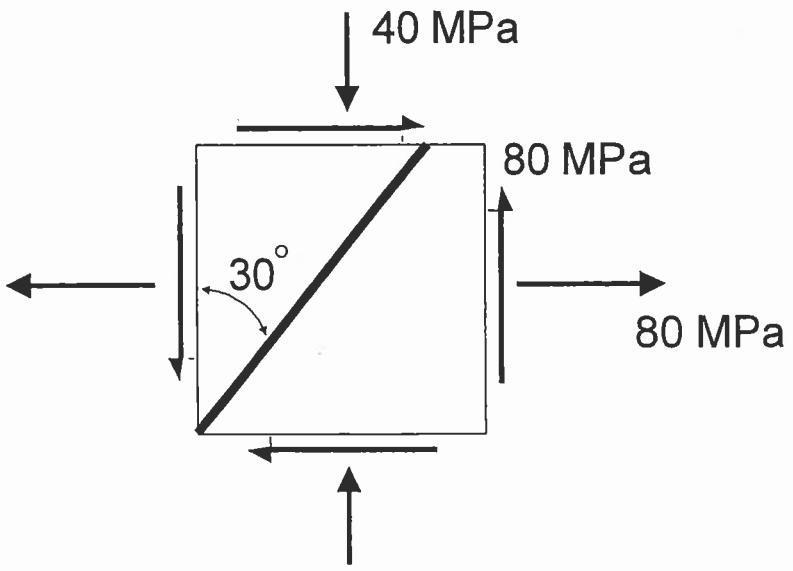
beam cross-section
(all dimensions in mm)

Question 6: The state of stress is shown below for a welded plate, with the weld making an angle of 30° with the vertical axis as indicated.

[20 marks] Use the Mohr's circle solution (*not* the transformation equations) to determine the following:

(a) the stress components acting on the weld, showing your answer on a properly oriented element.

(b) the maximum in-plane shear stress of the plate (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.

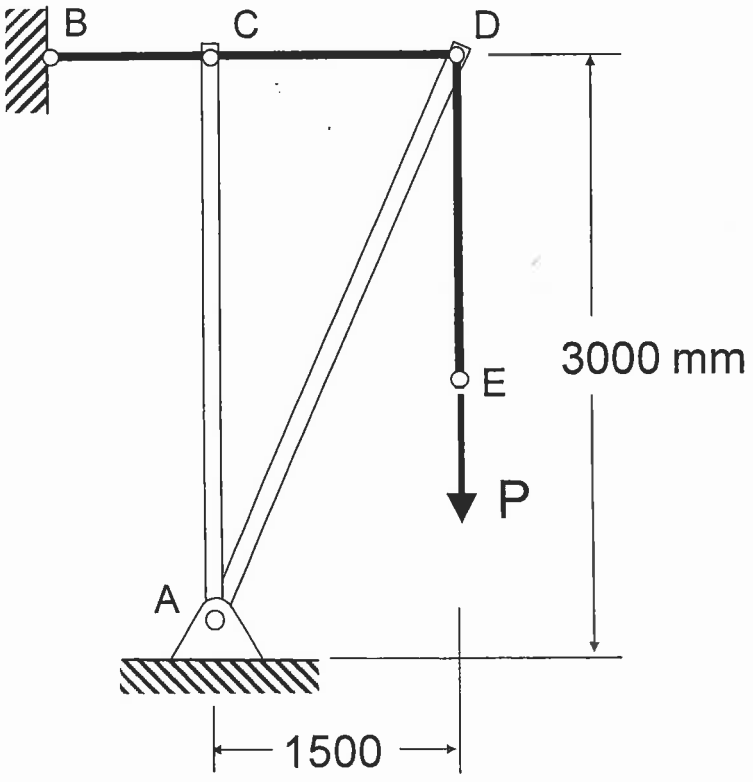


Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 7: The structure below is made from two steel rods (AC and AD) connected together with 40 mm diameter steel cables (BC, CD, and DE). Both steel rods have a diameter of 100 mm, and are considered pinned at their ends.

[20 marks] Determine the maximum load P that can be applied to this system as shown.

Consider in-plane buckling only for the compression members and use a factor of safety of 2.5 for the Euler buckling load. The steel used in the boom rods (AC and AD) has an elastic modulus of E = 200 GPa and an allowable yield strength equal to 240 MPa. The steel cable has a much higher yield strength of 1100 MPa and elastic modulus of E = 200 GPa.

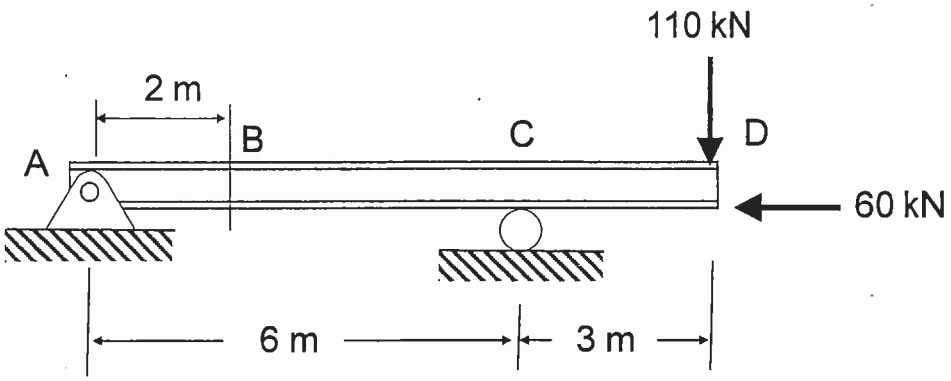


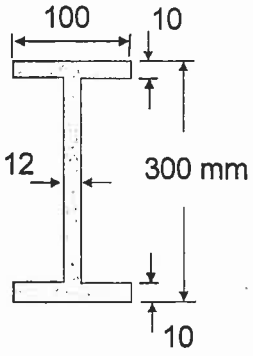
Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$

Question 8: [20 marks] A wide flange beam is loaded as shown below, with the pinned connection at A located at the centroid of the member. The 60 kN horizontal force at D is offset from the member centroid by 150 mm.

(a) Compute and sketch the distribution of normal force at section B located 2 m from support A.

(b) What is the maximum shear stress in the beam at B?





beam cross-section
(all dimensions in mm)