NATIONAL EXAMINATIONS

December 2010

04-BS-7 MECHANICS OF FLUIDS

Three (3) hours duration

Notes to Candidates

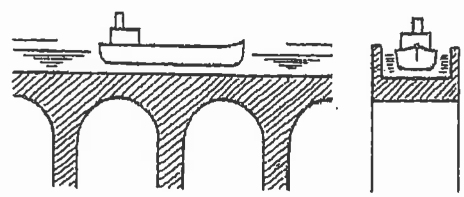
1. This is a **Closed Book** examination.
2. Exam consists of two Sections. **Section A is Analytical (5 questions)** and **Section B is Calculative (9 questions).**
3. **Do three (3) questions from Section A (Analytical) and seven (7) questions from Section B (Calculative).** Note that the Analytical Questions do not require detailed calculations.
4. **Ten (10) questions constitute a complete paper.** (Total 50 marks).
5. **All questions are of equal value.** (Each 5 marks).
6. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
7. Candidates may use one of the approved Casio or Sharp calculators.
8. **Reference data** for particular questions are given on pages 8 to 10. **These pages are to be returned with the Answer Booklet.**
9. **Constants** are given on page 11.
10. **Reference Equations** are given on pages 12 to 15.

SECTION A ANALYTICAL QUESTIONS

<u>Do three of five questions.</u> These questions do not require detailed calculations but complete written explanations must be given to support the answers. Where appropriate mathematical derivations or calculations should be given to justify the conclusions.

QUESTION 1

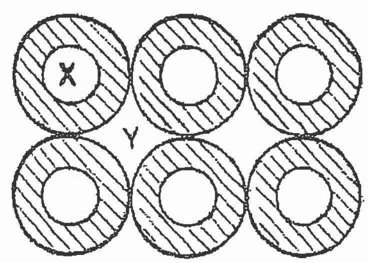
A barge 15 m long and 3 m wide is loaded such that its draught (depth of bottom below water surface) is 1.2 m. It sails in a long canal 5 m wide and 2 m deep. At one point the canal is taken across a valley by an aqueduct as shown in the adjacent sketch. Determine the <u>change</u> in compressive force on the aqueduct pillars as the barge passes over them. Give a full explanation of your answer.



(5 marks)

QUESTION 2

Small glass tubes are packed vertically in a square array as shown in the adjoining sketch. The inside diameter of the tubes is one half that of the outside diameter. The bottom of the array is immersed in water. State whether the water will rise to a greater height inside the tubes (space X) or between the tubes (space Y). Explain your statement using appropriate mathematical logic.



Surface tension of water $\sigma = 0.0728$ N/m
Contact angle with glass $\theta = 0°$

If necessary, to facilitate the explanation and mathematical logic, a small diameter for the tubes may be assumed.

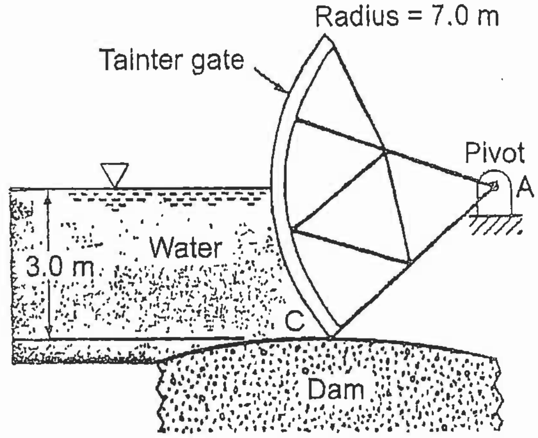
(5 marks)

QUESTION 3

A gate in the form of a partial cylindrical surface (called a *Tainter gate)* holds back water on top of a dam as shown in the adjacent figure. The radius of the surface in contact with the water is 7 m, and its length along the top of the wall is 11 m. The gate can pivot about point *A*, and the pivot point, which is at the centre of the circle forming the surface, is 3 m above the seat, *C*.

(a) State whether the resultant force will pass through the pivot. Explain the answer.

(b) Explain also how the horizontal and vertical components of the force on the gate could be calculated.



(5 marks)

QUESTION 4

For fully developed pipe flow describe the difference between laminar and turbulent flow. Clarify the conditions under which laminar and turbulent flow occur. By means of sketches illustrate the velocity profiles for the two cases. Explain why the two velocity profiles are different. In particular note the difference between the average and the maximum velocities in each case.

(5 marks)

QUESTION 5

Explain with the aid of sketches how lift is created by an airfoil. The explanation must refer to the fundamental principles of energy and momentum. Describe how lift and drag vary with angle of attack.

(5 marks)

SECTION B CALCULATIVE QUESTIONS

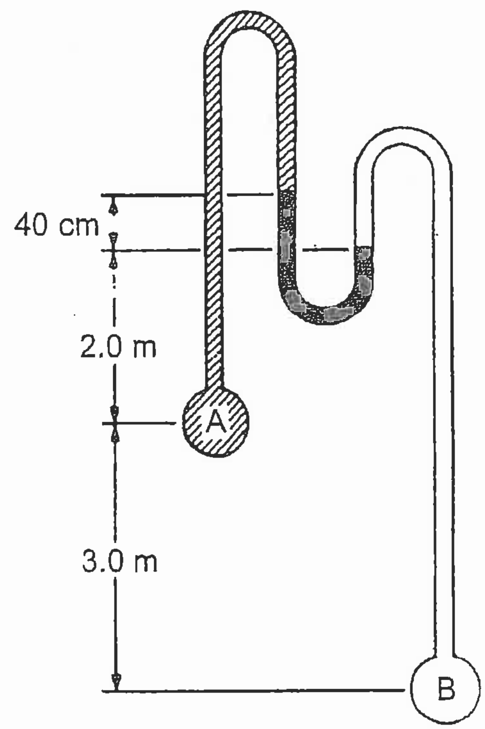
Do seven of nine questions. ***Solutions to these questions must be set out logically with all intermediate answers and respective units given.***

QUESTION 6

Refer to the adjoining illustration. Use the differential elevations in metres and centimetres as given in the figure. Pipe A contains benzene and pipe B contains carbon tetrachloride while the U-tube contains mercury. Determine the pressure in pipe A if the pressure in pipe B is 200 kPa.

Refer to Constants on page 11 for specific gravities.

(5 marks)

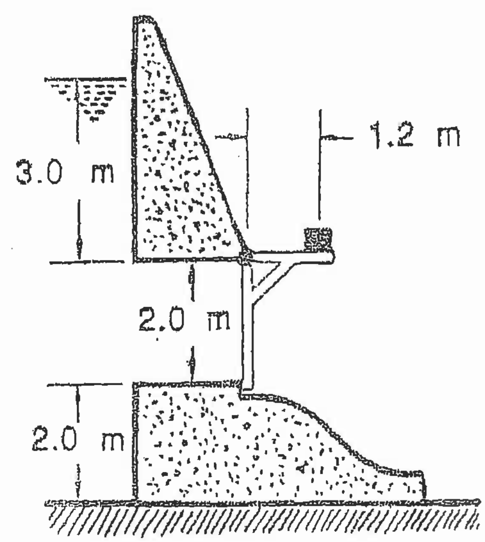


QUESTION 7

The adjoining sketch shows a hinged gate in the wall of a dam. The gate is rectangular being 1.5 m wide and 2.0 m high. It is located so that the top of the gate is 3.0 m below the normal water level. It is held closed by a weight located 1.2 m from the hinge. Calculate the required mass for the weight such that the gate will begin to open on rising water level. Neglect the mass of the gate.

Note: Moment of Inertia of a Rectangle about its centroid is $I_c = (bh^3/12)$

(5 marks)



QUESTION 8

Water is pumped through a pipeline to supply the needs of a certain community. The pipe is 400 mm in diameter and 4800 m long. The flow rate is 0.36 m^3/s. It is suspected that corrosion in the pipe has caused excessive head loss so measurements are made at each end of the pipe. Near the pump the pressure is 2320 kPa and the elevation 84 m. Near the discharge the pressure is 120 kPa and the elevation 166 m. Calculate the head loss in the pipe.

(5 marks)

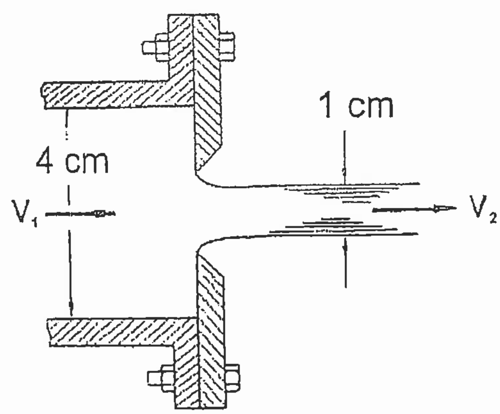
QUESTION 9

A venturi tube carrying water is mounted in a vertical position with flow downwards. Pressure tappings at the inlet and at the throat are connected to a water-mercury manometer. The inlet diameter is 80 cm and the throat diameter is 40 cm and the distance apart of these points (also the vertical distance between pressure tappings) is 200 cm. If the reading on the manometer is 15 cm between mercury levels, find the flow rate of the water. Assume ideal flow conditions, that is, no friction loss.

(5 marks)

QUESTION 10

A high velocity water jet is created by attaching a plate with a sharp edged orifice to the end of a pipe 4 cm in diameter. The pressure in the pipe is 3 MPa gauge.



(a) Calculate the velocity in the pipe and in the 1 cm diameter jet so created and hence the flow rate from the nozzle.

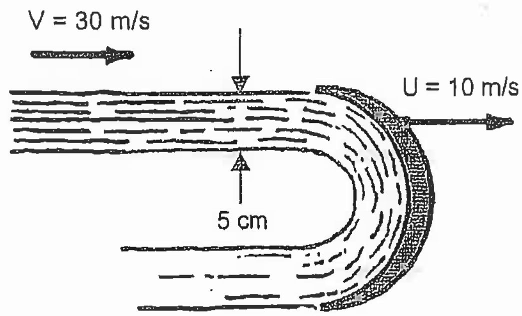
(b) Calculate the total force in the bolts needed to hold the orifice plate in place.

Assume ideal flow conditions, that is, no fluid friction.

(5 marks)

QUESTION 11

A jet of water with a diameter of 5 cm and a velocity of 30 m/s strikes a curved plate and is turned completely through 180° without friction loss. The plate is driven at 10 m/s by the jet and in the same direction as the jet.



(a) Calculate the force exerted on the plate.
(b) Calculate the power in the jet.
(c) Calculate the power developed by the plate.
(d) Determine the efficiency of energy transfer.

(5 marks)

QUESTION 12

A 250 mm diameter pipeline is 5 km long and it has an absolute roughness, ε, of 0.25 mm. Water at a flow rate of 0.1 m^3/s is to be pumped through it with a total actual lift of 6 m. Calculate the head loss due to friction and the power required to pump the water.

Use the attached Moody Diagram on page 9 to obtain the necessary data.

(5 marks)

QUESTION 13

A commercial steel pipeline is required to convey water from a storage reservoir to a local supply head tank. The length of the pipeline is 5 km and the difference in head is 120 m. Select a suitable pipe diameter to give a flow rate of 1 m^3/sec.

Refer to the attached Pipe Flow Data and Moody Diagram on pages 8 and 9 respectively. Plot assumed points on the Moody Diagram to show how the answer is obtained. Return this diagram with the answer booklet.

(5 marks)

QUESTION 14

A steel sphere with a specific gravity of 7.8 and a diameter of 6 mm is released in a large tank of oil with a specific gravity of 0.825. The sphere is observed to have a terminal velocity of 0.5 m/s. Calculate the absolute viscosity of the oil.

Refer to the attached Drag Coefficients for Spheres on page 10 for empirical data.

(5 marks)

QUESTION 13 PIPE FLOW DATA

TABLE 8.3 **Values of loss factors for pipe fittings**

Fitting	k	L/D
Globe valve, wide open	10	350
Angle valve, wide open	5	175
Close-return bend	2.2	75
T, through side outlet	1.8	67
Short-radius elbow	0.9	32
Medium-radius elbow	0.75	27
Long-radius elbow	0.60	20
45° elbow	0.42	15
Gate valve, wide open	0.19	7
half open	2.06	72

TABLE 8.2 **Loss coefficients for sudden contraction**

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k_c	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

TABLE 8.1 **Values of absolute roughness e for new commercial pipes**

	Feet	Millimeters
Glass, plastic (smooth)	0.0	0.0
Drawn tubing, brass, lead, copper, centrifugally spun cement, bituminous lining, transite	0.000 005	0.0015
Commercial steel, wrought iron, welded-steel pipe	0.00015	0.046
Asphalt-dipped cast iron	0.0004	0.12
Galvanized iron	0.0005	0.15
Cast iron, average	0.00085	0.25
Wood stave	0.0006–0.003	0.18–0.9
Concrete	0.001–0.01	0.3–3
Riveted steel	0.003–0.03	0.9–9

Note: $\dfrac{e}{D} = \dfrac{e \text{ in feet}}{D \text{ in feet}} = 12 \times \dfrac{e \text{ in feet}}{D \text{ in inches}} = \dfrac{e \text{ in mm}}{D \text{ in mm}}$

QUESTION 12 AND QUESTION 13 MOODY DIAGRAM

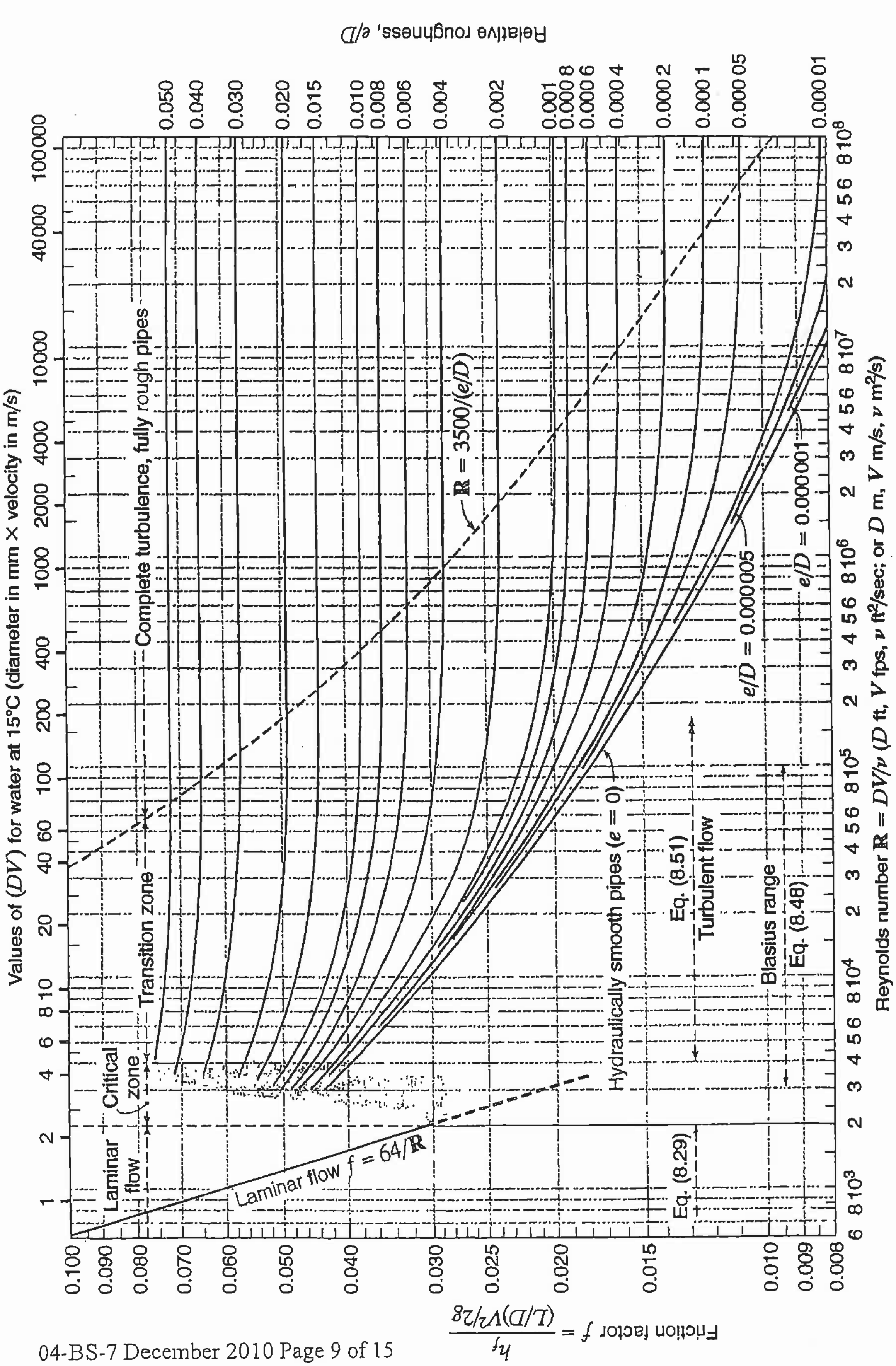


Figure 8.11
Moody chart for pipe friction factor (Stanton diagram).

QUESTION 14 DRAG DIAGRAM FOR SOLID BODIES

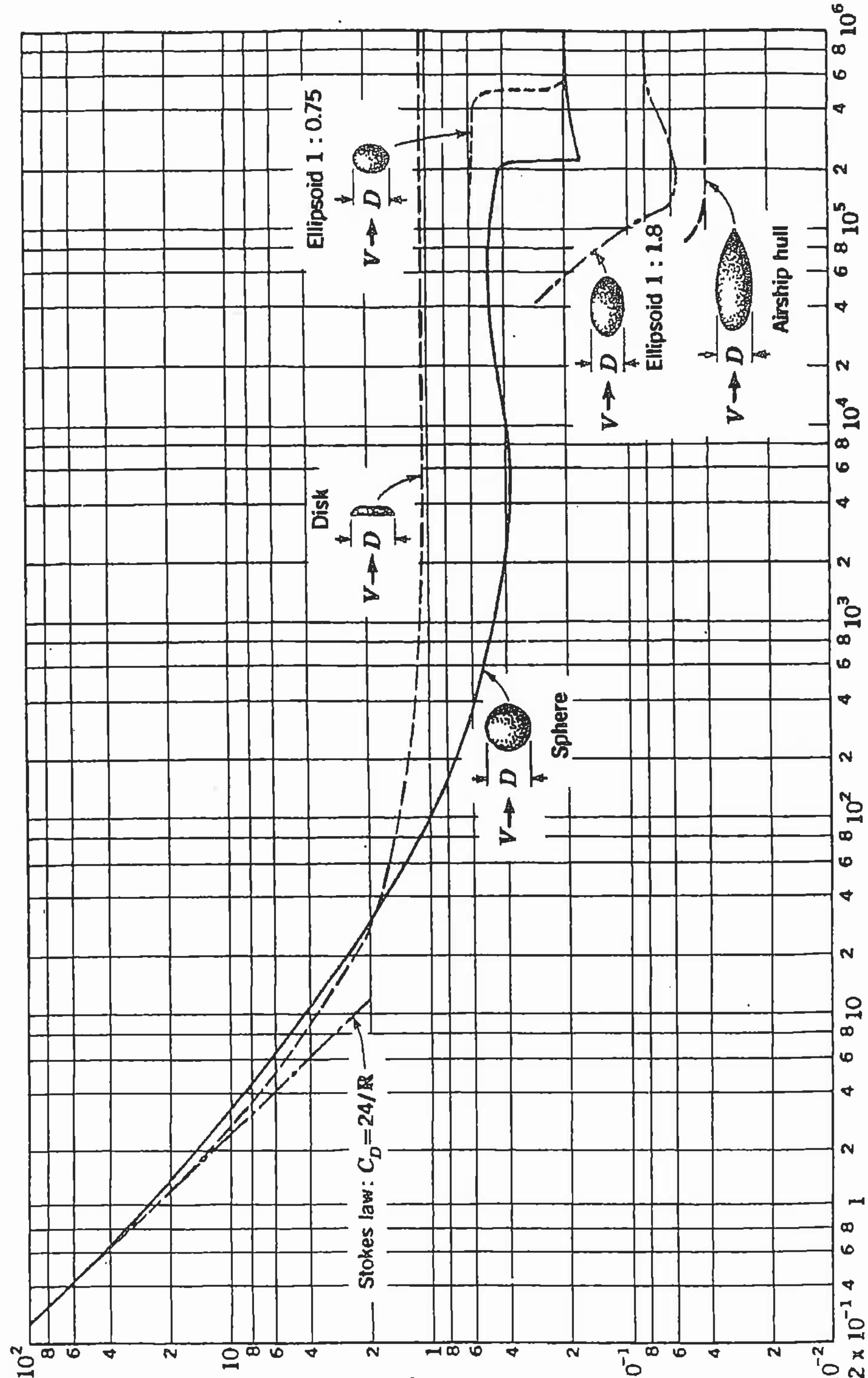


Figure 10.10 Drag coefficient for bodies of revolution. (Adapted from L. Prandtl, "Ergebnisse der aerodynamischen Versuchsanstalt zu Göttingen," p. 29, R. Oldenbourg, Munich and Berlin, 1923; and F. Eisner, "Das Widerstandsproblem," *Proc. 3d Internatn. Congr. Appl. Mech., p. 32, 1930.*)

04-BS-7 MECHANICS OF FLUIDS

GENERAL REFERENCE INFORMATION

CONSTANTS

In engineering calculations a high degree of accuracy is seldom attained due to the neglect of minor influences or the inaccuracy of available data. For consistency in calculations however the following reasonably accurate constants should be used:

Atmospheric Pressure p_o = 100 kPa
Specific Gravity of Water = 1.00
Specific Gravity of Glycerine = 1.26
Specific Gravity of Mercury = 13.56
Specific Gravity of Benzene = 0.90
Specific Gravity of Carbon Tetrachloride = 1.59
Density of Water ρ = 1000 kg/m^3
Density of Sea Water ρ = 1025 kg/m^3
Density of Concrete ρ = 2400 kg/m^3
Density of Air ρ = 1.19 kg/m^3 (at 20°C), ρ = 1.21 kg/m^3 (at 15°C)
Absolute Viscosity of Water μ = 1.0 x 10^{-3} Ns/m^2
Absolute Viscosity of Air μ = 1.8 x 10^{-5} Ns/m^2
Gravitational Acceleration g = 9.81 m/s^2
Surface Tension of Water σ = 0.0728 N/m (at 20°C)
Specific Heat of Water c_p = 4.19 kJ/kg°C
Specific Heat of Air c_p = 1005 J/kg°C
Specific Heat of Air c_v = 718 J/kg°C
Gas Constant for Air R = 287 J/kg°K
Gas Constant for Helium R = 2077 J/kg°K
Gas Constant for Hydrogen R = 4120 J/kg°K

REFERENCE EQUATIONS

Equation of State

$$p\,v = R\,T$$

$$p = \rho\,RT$$

Universal Gas Law

$$p\,v^n = \text{constant}$$

Compressibility

$$\beta = -\Delta / V\,\Delta p$$

Viscous Force and Viscosity

$$F = \mu\,A\,du/dy$$
$$\mu = \tau\,du/dy$$
$$\nu = \mu/\rho$$

Capillary Rise and Internal Pressure due to Surface Tension

$$h = (\sigma \cos\theta / \rho\,g) \times (\text{perimeter} / \text{area})$$
$$p = 2\,\sigma / r$$

Pressure at a Point

$$p = \rho\,g\,h$$

Forces on Plane Areas and Centre of Pressure

$$F = \rho\,g\,y_c\,A$$
$$y_p = y_c + I_c / y_c\,A$$

Moments of Inertia

Rectangle: $I_c = b\,h^3 / 12$
Triangle: $I_c = b\,h^3 / 36$
Circle: $I_c = \pi\,D^4 / 64$

Volumes of Solids

Sphere: $V = \pi D^3 / 6$

Cone: $V = \pi D^2 h / 12$

Spherical Segment: $V = (3a^2 + 3b^2 + 4h^2) \pi h / 2g$

Continuity Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = M$$

General Energy Equation

$$p_1 / \rho_1 g + z_1 + V_1^2 / 2g + q_{in} / g + w_{in} / g$$

$$= p_2 / \rho_2 g + z_2 + V_2^2 / 2g + h_L + q_{out} / g + w_{out} / g$$

Bernoulli Equation

$$p_1 / \rho g + z_1 + V_1^2 / 2g = p_2 / \rho g + z_2 + V_2^2 / 2g$$

Momentum Equation

Conduit: $F_R = p_1 A - p_2 A - M (V_2 - V_1)$

Free Jet: $F_R = -\rho Q (V_2 - V_1)$

Flow Measurement

Venturi Tube: $Q = [C A_2 / \{1 - (D_2 / D_1)^4\}^{1/2}][2 g \Delta h]^{1/2}$

Flow Nozzle: $Q = K A_2 [2 g \Delta h]^{1/2}$

Orifice Meter: $Q = K A_o [2 g \Delta h]^{1/2}$

Flow over Weirs

Rectangular Weir: $Q = C_d (2/3) [2g]^{1/2} L H^{3/2}$

Power

Turbomachine: $P = \rho g Q H$

Free Jet: $P = ½ \rho Q V^2$

Moving Blades: $P = M \Delta V U$

Steam Turbine: $P = M (V_{S1} \cos\theta - V_{S2} \cos\delta) U$

Aircraft Propulsion

$$F_{thrust} = M\,(V_{jet} - V_{aircraft})$$
$$P_{thrust} = M\,(V_{jet} - V_{aircraft})\,V_{aircraft}$$
$$E_{jet} = \tfrac{1}{2}\,(V_{jet}^2 - V_{aircraft}^2)$$
$$P_{jet} = \tfrac{1}{2}\,M\,(V_{jet}^2 - V_{aircraft}^2)$$
$$E_{fuel} = CV_{fuel}$$
$$P_{fuel} = M_{fuel}\,CV_{fuel}$$
$$\eta_{thermal} = P_{jet} / P_{fuel}$$
$$\eta_{propulsion} = P_{thrust} / P_{jet} = 2\,V_{aircraft} / (V_{jet} + V_{aircraft})$$
$$\eta_{overall} = \eta_{thermal} \times \eta_{propulsion}$$

Wind Power

$$P_{total} = \tfrac{1}{2}\,\rho\,A_T\,V_1^3$$
$$P_{max} = 8/27\,\rho\,A_T\,V_1^3$$
$$H_{max} = P_{max} / P_{total} = 16/27$$

Reynolds Number

$$Re = d\,V\,\rho / \mu$$

Flow in Pipes

$$h_L = f\,(L / D)\,(V^2 / 2\,g)$$
$$D_e = 4\,(\text{flow area}) / (\text{wetted perimeter})$$
$$D = D_e \qquad \text{for non-circular pipes}$$
$$L = L_{total} + L_e \qquad \text{for non-linear pipes}$$
$$(L / D) = 35\,k \qquad \text{for } Re \sim 10^4$$

Pump Specific Speed and Net Positive Suction Head

$$N_s = \omega\,Q^{1/2} / (g\,H)^{3/4}$$
$$u_2 = \phi\,(2\,g\,H)^{1/2}$$
$$u_2 = \pi\,D\,N / 60$$
$$\sigma_c = NPSH_{required} / H$$
$$NPSH_{available} = (p_{atmosphere} - p_{vapour}) / (\rho\,g) - z - h_L$$

Drag on Immersed Bodies

Friction Drag:	$F_f = C_f \frac{1}{2} \rho V^2 B L \quad (B = \pi D)$
Pressure Drag:	$F_p = C_p \frac{1}{2} \rho V^2 A$
Total Drag:	$F_D = C_D \frac{1}{2} \rho V^2 A$
Aircraft Wing:	$F_L = C_L \frac{1}{2} \rho V^2 A_{wing}$
Aircraft Wing:	$F_D = C_D \frac{1}{2} \rho V^2 A_{wing}$

Karmen Vortex Frequency

$$f \approx 0.20 \, (V / D) \, (1 - 20 / Re)$$