National Exams December 2010

04-CHEM-B1, Transport Phenomena

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an OPEN BOOK EXAM.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 marks. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

Section A: Fluid Mechanics

A1 **[20 marks overall]** An incompressible viscous fluid undergoes steady laminar flow through a circular pipe of length L and radius R.

(a) **[15 marks]** Starting with the following form of the Navier-Stokes equation

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \nu \frac{\partial^2 u_z}{\partial r^2} + \frac{\nu}{r}\frac{\partial u_z}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \nu \frac{\partial^2 u_z}{\partial z^2}$$

show through clear assumptions and logical analysis that the relationship of the axial velocity at any point (u_z) to the maximum velocity ($u_{z,max}$) is:

$$\frac{u_z}{u_{z,\max}} = 1 - \frac{r^2}{R^2}$$

(b) **[5 marks]** Also show that the volumetric flow rate is given by:

$$\dot{V} = \pi\left(\frac{-\Delta P}{8\mu L}\right)R^4$$

A2 **[25 marks overall]** A decrease of 0.71 m in head of liquid is required to drive a liquid (density = 984 kg/m^3) through a smooth horizontal pipeline (inside diameter = 0.55 m, length = 658 m) at a volumetric flow rate of 0.206 m^3/s. Calculate the following:

(a) **[5 marks]** The pressure drop per unit length of pipe in Pa/m.

(b) **[5 marks]** The Fanning friction factor.

(c) **[5 marks]** The total force exerted by friction on the pipeline.

(d) **[5 marks]** The kinematic viscosity of the liquid in cSt.

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Section B: Heat Transfer

B1 **[25 marks overall]** Consider a hollow cylindrical heat transfer medium of length L having inside and outside radii of r_i and r_0 with corresponding temperatures T_i and T_0 . If the thermal conductivity varies linearly with temperature according to:

$$k(T) = k_0(1+\beta T)$$

starting with the appropriate form of Fourier's law of heat conduction show that the steady-state rate of heat transfer in the radial direction $\dot{Q}_r$ is given by:

$$\dot{Q}_r = \frac{2\pi L k_0}{\ln(r_0/r)}\left[1+\frac{\beta}{2}(T_i+T_0)\right]\cdot(T_i-T_0)$$

B2 **[25 marks overall]** A large slab of pure aluminium is of half-thickness L = 0.5 m. It is initially at a uniform temperature of 200°C and is suddenly exposed to a convection environment in which the fluid temperature is 50°C and the heat transfer coefficient is 948 W/m^{2}K. Calculate the following:

(a) **[15 marks]** The temperature of the slab 200 mm from one of the surfaces after 30 minutes; and

(b) **[10 marks]** How much heat is lost from the slab during this time?

Useful charts are shown in Figs B1 and B2.

Useful data for aluminium:

ρ = 2,702 kg/m^3; c_P = 903 J/kg K; k = 237 W/ m K; and α = 97.1x10^{-6} m^2/s

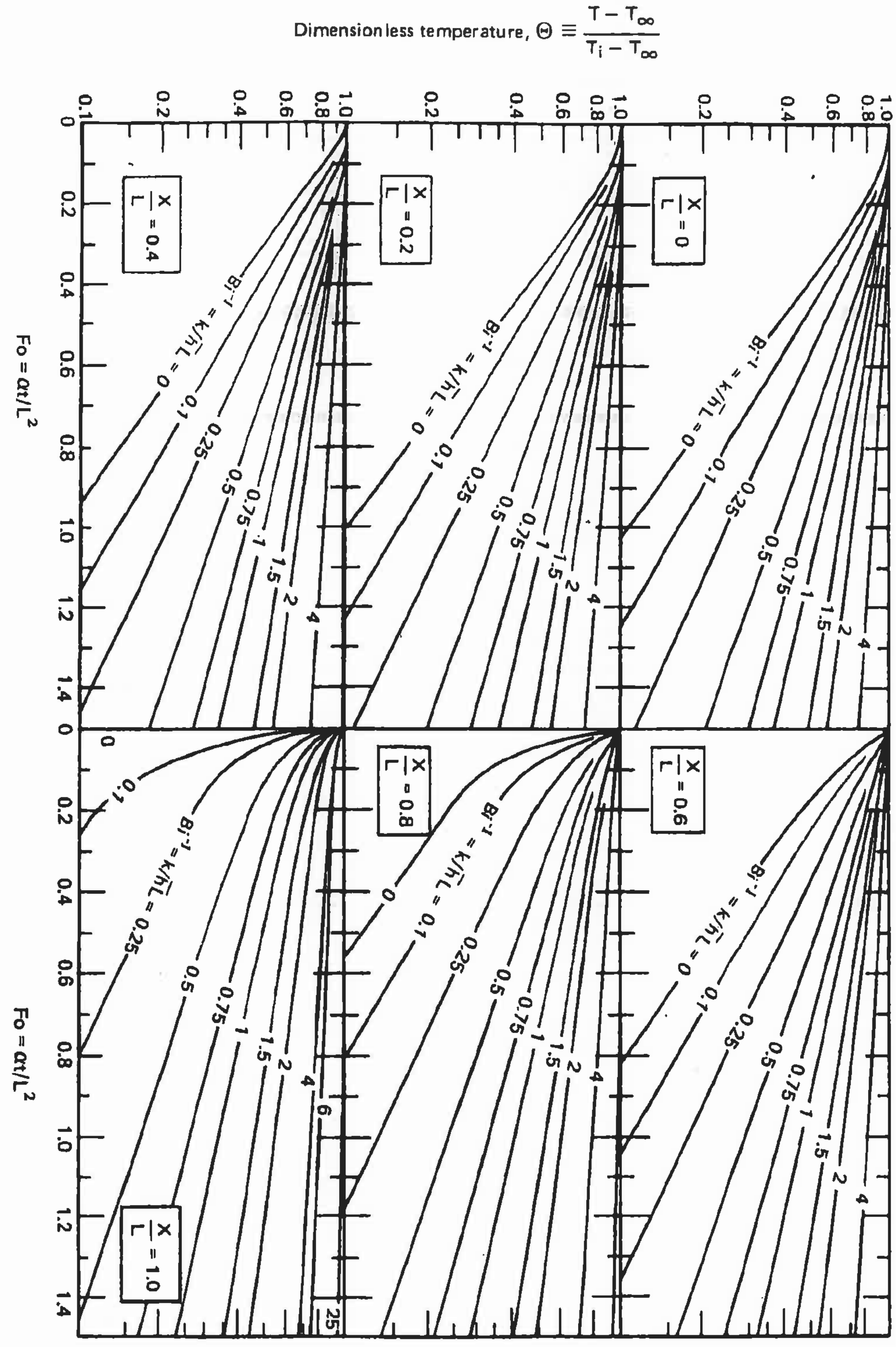
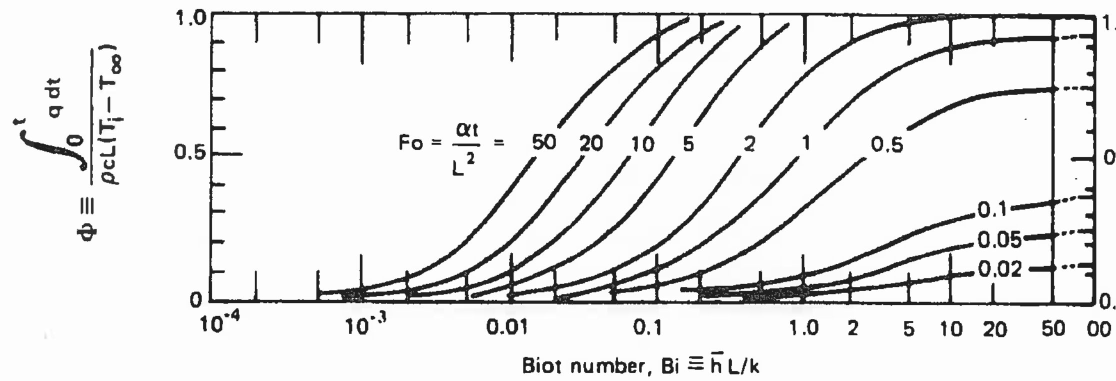
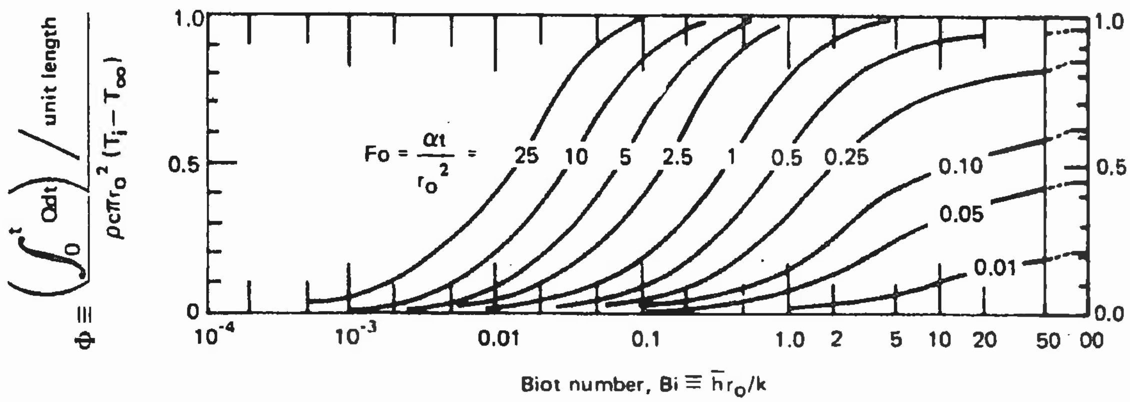


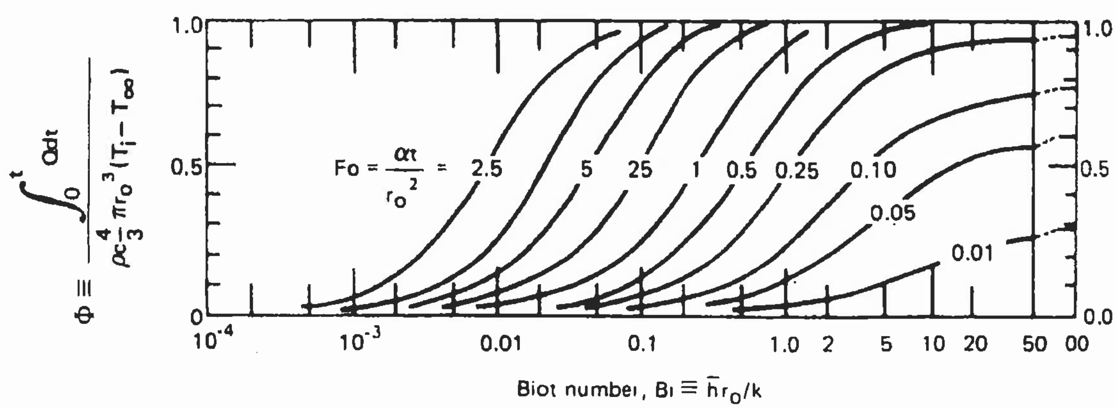
Fig. B1: The transient temperature distribution in a slab at six positions; *x/L = 0* is the centre, and *x/L = 1* is the outside boundary.



a.) *Slab* of thickness, L, insulated on one side



b.) *Cylinder*, of radius, r_o



c.) *Sphere*, of radius, r_o

Fig. B2: The heat removal from suddenly cooled bodies as a function of $\bar{h}$ and time

Section C: Mass Transfer

C1 [25 marks overall] A permeable thick-walled hollow sphere has an internal radius of R_1 and an external radius of R_2. The inner surface is maintained at a constant concentration C_{Ai} and the outer surface is maintained at a constant concentration C_{Ao}. Starting with the following form of the continuity equation

$$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi} =$$

$$\dot{C}_{A,G} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D \frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta \frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right)$$

show through clear assumptions and logical analysis that the steady-state concentration profile for component A within the wall is:

$$C_A = C_{Ai} + \left\{(C_{Ao} - C_{Ai})R^2\left[(r - R_1)/r\right]/(R_2 - R_1)\right\}$$

C2 [25 marks overall] A piece of porous glass tubing is used as a diffusion cell to measure the diffusion coefficient of an air-gas mixture. The inside diameter and the outside diameter of the cell are 1 mm and 4 mm. It was found that with a difference in mole fraction of 10% at 25°C and 1 atmosphere pressure, the molar flow rate was 4×10^{-6} mol/s per cm of length. Find the diffusion coefficient in m^2/s.