National Examinations December 2010

07-Elec-A3 Signals and Communications

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2. This is a Closed Book Exam. No aids other than an approved Casio or Sharp calculator is permitted.

3. There are six questions in total, and any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1. An AM modulator operates with the message signal

$$m(t) = 9\cos(20\pi t) + 8\cos(60\pi t).$$

The unmodulated carrier is given by $110\cos(200\pi t)$, and the system operates with a modulation index of 0.5.

(a) Find the period of $m(t)$.

(b) Find the maximum value and the minimum value of $m(t)$.

(c) Derive the expression representing the modulated signal, denoted by $x(t)$.

(d) Using the result of part (b), determine the power efficiency of this particular AM modulation process.

2. An FM modulator has output

$$x(t) = 100\cos\left[2\pi f_c t + 2\pi f_d \int_0^t m(\alpha)d\alpha\right]$$

where $f_d = 20$ Hz/V. Assume that $m(t)$ is the rectangular pulse

$$m(t) = \begin{cases} 4, & 0 < t \leq 8 \\ 0, & \text{elsewhere} \end{cases}.$$

(a) Sketch the phase deviation in radians.

(b) Sketch the frequency deviation in Hz.

(c) Using Carson's Rule, determine the approximate bandwidth of $x(t)$.

(d) Find the power of $x(t)$.

3. (a) Show that the Fourier transform $X(j\omega)$ or $X(f)$ of a signal with even symmetry i.e., $x(t) = x(-t)$, is real-valued.

   (b) Find the Fourier transform of the signal

$$x(t) = \begin{cases} 5 & -\frac{T}{2} < t \leq \frac{T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

   and sketch its amplitude spectrum.

   (c) Consider the signal

$$y(t) = \begin{cases} 2t & 0 < t \leq 2 \\ -2t + 8 & 2 < t \leq 4 \\ 0 & \text{elsewhere} \end{cases}.$$

   $y(t)$ can be derived from $x(t)$ using time-shifting, convolution and amplitude scaling with an appropriate choice of T. Using the time-shift, convolution and linearity properties of the Fourier transform, find the Fourier transform of $y(t)$.

4. (a) Consider the pulse train described by

$$p(t) = \begin{cases} \cos(10\pi t) & -0.05 + k < t < 0.05 + k \\ 0 & \text{elsewhere} \end{cases}$$

   where k is any integer. Note that $p(t)$ has a period of 1 second. Find $P(j\omega)$, the Fourier transform of $p(t)$. (*Hint:* It is a train of modulated impulses.)

   (b) The spectrum of a signal $x(t)$ is

$$X(j\omega) = \begin{cases} 1 & -\frac{\pi}{2} < \omega < \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

   and it is multiplied by $p(t)$ from part (a) to give $y(t) = x(t)p(t)$. Can $x(t)$ be perfectly recovered from $y(t)$? If so, describe briefly how.

5. A passband digital communication transmitter uses quaternary phase shift keying (QPSK), a carrier frequency of 100 MHz, average symbol power of 1 mW, and a symbol (or baud) rate of 10 kHz. It uses a transmit pulse denoted by $p(t)$.

   (a) Sketch a signal constellation (i.e. the set of points in signal space that a symbol can take) corresponding to the above description.

   (b) If $p(t) = 1$ for $0 < t \leq T$, and 0 elsewhere, where T is the symbol interval, write down an expression for the transmitted signal in the time domain. Use the parameter values given above and the signal constellation presented in part (a).

   (c) What is the minimum bandwidth required for transmission with no inter-symbol interference? What is the pulse $p(t)$ that must be applied to achieve the minimum bandwidth?

   (d) Find the transmitted bit rate, and the bandwidth efficiency in bps/Hz, assuming that $p(t)$ is the minimum-bandwidth Nyquist pulse.

6. Consider the signal $g(t) = \text{sinc}(5t)\text{sinc}(2t)$. We define

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t}$$

   and note that the Fourier transform of $B\text{sinc}(Bt)$ is $\Pi(f/B)$ where $\Pi(f) = 1$ for $-0.5 < f \leq 0.5$ and 0 elsewhere.

   (a) Find the energy spectral density (ESD) of $g(t)$.

   (b) From part (a), find the Nyquist sampling rate for $g(t)$.

   (c) Sketch the spectrum of the signal

$$g_\delta(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

   where $\delta(t)$ is the Dirac delta function (or impulse), and $1/T = 5$ Hz.