NATIONAL EXAMS Dec 2010
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio FX-991 or a Sharpe EL-540.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

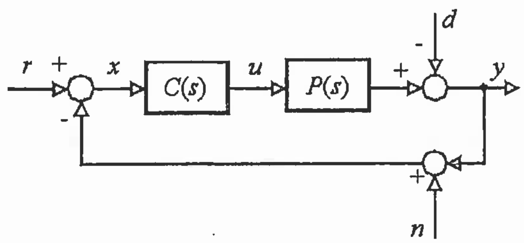
5. All questions are of equal value.

1. Consider the control system below with, $P(s) = \dfrac{1-s}{(1+s)^2}$.

(a) Design a proportional controller, $C(s)$, such that the phase margin is 45 degrees. What is the corresponding gain margin?

(b) Determine the steady state value of x when $r = 10$ and $d = 3$ and $n = 0$.

(c) Determine the steady state value of x when $r = 0$ and $d = 3$ and $n = 10\cos(5t)$.



2. Consider the system, $y(s) = G(s)u(s)$, $G(s) = \dfrac{\alpha - \beta s}{(1+s)^2}$.

(a) Find a state space model for the system taking $y(t)$ and $\dot{y}(t)$ as state variables.

(b) Justify the conditions under which the system is controllable and observable.

(c) Let $\alpha = \beta = 1$. Design a state feedback controller such that the closed loop poles are -10, -5.

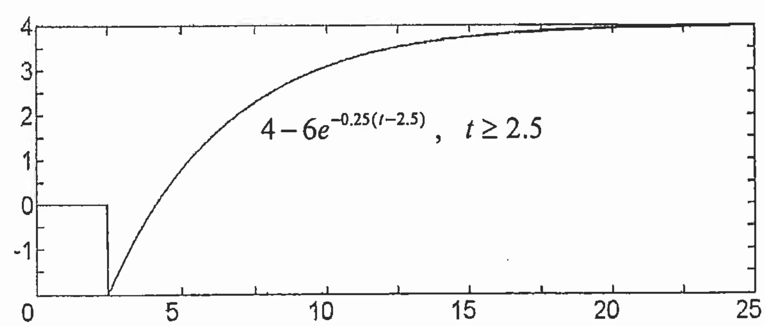
3. Consider the system, $P(s) = \dfrac{1-s}{(1+s)^2}$.

(a) Is it possible to design a PI controller such that the gain crossover frequency is much greater than 1 rad/s? Explain why or why not.

(b) Design a PI controller such that the gain margin is 2 and the gain crossover frequency is as large as feasible.

(c) Determine the closed loop transfer function that relates the set point input to the output and sketch approximately the unit step response.

4. A unit step is applied at the input of an open loop plant, $P(s)$, at time $t = 0$. The measured response is shown on the right.



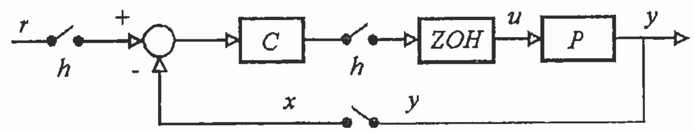
(a) Establish whether the response exhibits non-minimum phase behavior and/or a transportation delay.

(b) Determine the transfer function, $P(s)$.

(c) A sinusoid, $3\sin(4t)$ is applied to the input of $P(s)$. Determine the steady state output.

(d) Outline the precise steps and conditions necessary to establish closed loop stability when the system is controlled with unity negative feedback.

5. Consider the sampled data system shown on the right. The input to the ZOH, the set-point, r, and the output, y, are uniformly sampled with a sample period of h. $C(z)$ and $P(s)$ are given by,

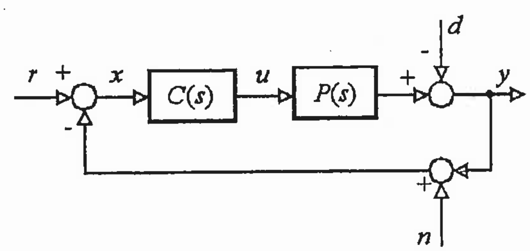


$$C(z) = Kz^{-1}, \quad P(s) = \frac{1}{s+2}$$

(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.
(b) Determine the range of K for stability.
(c) Sketch the associated root locus.

6. Consider the feedback system below with, $C(s) = \frac{K}{s}, \quad P(s) = \frac{e^{-s}}{s+1}$.

(a) Determine the range of K that results in closed loop stability.
(b) For $K = 1$ sketch the associated Bode plot and determine the gain crossover frequency.
(c) The system is stable and operating with constant inputs, $n(t) = 0$, $d(t) = d_0$, and $r(t) = r_0$. Determine the steady state tracking error, $e = r - y$, as a function of K.
(d) Over what (approximate) range of frequencies is the effect of the noise, n, attenuated at y?



Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{\left(C+jD\right)z}{z-re^{j\varphi}}+\dfrac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{\left(z-a\right)^{r}}$, $r=2,3\ldots$	$\dfrac{Kn\left(n-1\right)\ldots\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^{n}$

Table of Laplace and z-Transforms (*h* denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos \beta h)}{z^2-2z\cos \beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin \beta h}{z^2-2z\cos \beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos \beta h)}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin \beta h}{z^2-2ze^{-\alpha h}\cos \beta h+e^{-2\alpha h}}$
$t f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$