National Exams December 2010

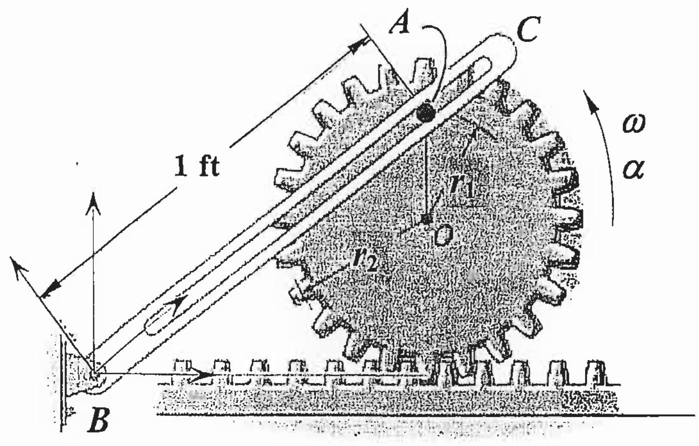
98-Phys-A1, Classical Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM.
   Casio or sharp approved calculator is permitted.
3. All FIVE (5) questions constitute a complete exam paper.
4. Each question's value is listed in the marking scheme at the end of the exam paper.

1. At the instant shown, the angular motion of the gear of radius $r_2 = 0.4\ ft$ has an angular velocity $\omega = 2\ rad/s$ and angular acceleration $\alpha = 1\ rad/s^2$. At this instant, the peg A, that is fixed to the gear and is at a distance $r_1 = 0.25\ ft$ from the center of the gear O, is also directly above the gear center O. Determine



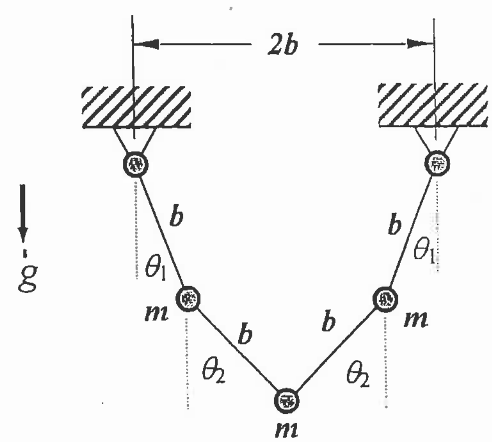
   (a) the angular velocity of the slotted link **BC**,

   (b) the angular acceleration of the slotted link **BC**, and

   (c) the velocity of the peg A traveling along the slot.

   Note: A Cartesian coordinate system attached at point B is provided for your reference. In addition, a pair of moving axes along and normal to the slotted bar is given at **B** for kinematics analysis using rotating axes.

2. The idea of virtual work is of fundamental importance in classical mechanics. For a system of N particles with virtual displacements $\delta\vec{x}_i$, $i = 1, \ldots N$, consistent to the constraints, the principle of virtual work reads $\delta V = \sum \vec{F}_i \bullet \delta\vec{x}_i = 0$, where $\vec{F}_i$ is the total forcing acting on the i^{th} particle and V is the potential energy function of the system. Now, consider the system of four identical, massless, rods of length b, connected via frictionless pin joints of mass m (see Figure). The distance between the supports is $2b$. Answer the following questions.



   (a) What is the constraint equation in θ_1 and θ_2.

   (b) Explain why the constraint is holonomic.

   (c) Determine the relationship between angles θ_1 and θ_2 in the static equilibrium.

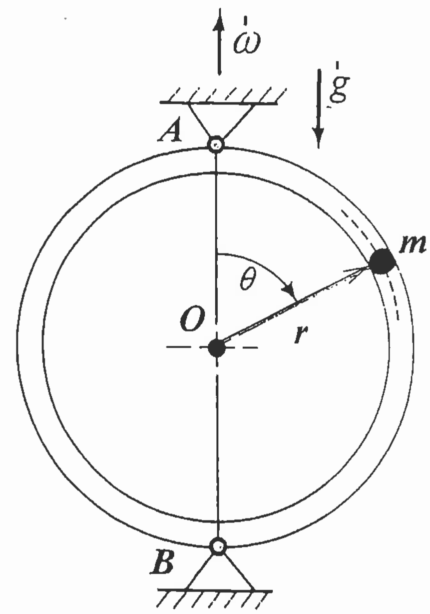
3. A particle of mass m can slide on a frictionless circular tube of radius r. The tube rotates about a vertical axis AB passing through the center of the tube O at a constant rate of ω *rad/s* (see Figure).

   (a) What is the Lagrangian *function* of the system?

   (b) What is the Lagrange's *equation*?

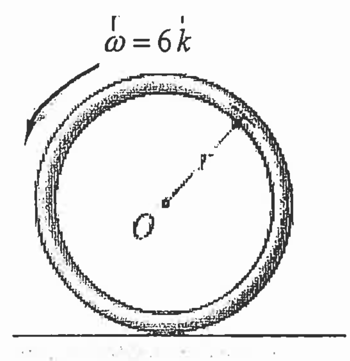
   (c) Derive the equations of motion.

   Note: Assume frictionless joints at A, B.



4. The thin ring of weight 10 *lb* and radius $r = 6$ *in* is given an initial angular velocity of 6 *rad/s* when it is placed on the surface. If the kinetic coefficient friction between the hoop and the surface is 0.3, determine the distance the hoop moves before it stops slipping.

Note: $\vec{k}$ denotes the direction perpendicular to the paper, $\vec{g} = 32.2\, ft/s^2$ and the moment of inertia of the ring rotating about the axis through its center O is $mr^2/2$.

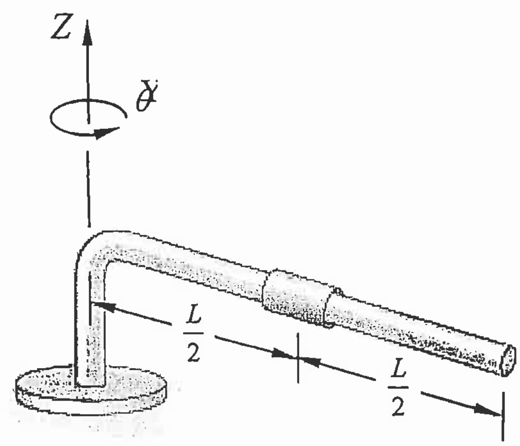


5. An L-shape rigid rod has a length L and mass m. A collar of one-fourth the mass of the rod is placed on the rod at its midpoint. Assume the rod is freely rotating at the angular speed $\dot{\theta}$ about the Z-axis when the collar is released. Answer questions (a) and (b) below assuming no friction between the collar and the rod.

   (a) Determine the rod's angular velocity just before the collar flies off the rod.

   (b) Determine the speed of the collar as it leaves the rod.

   Note: the moment of inertia of the rod rotating about the Z-axis is $mL^2/3$.



Marking Scheme

1. 15 marks total:
   (a) 5 marks
   (b) 5 marks
   (c) 5 marks

2. 25 marks total:
   (a) 10 marks
   (b) 5 marks
   (c) 10 marks

3. 25 marks total:
   (a) 10 marks
   (b) 10 marks
   (c) 5 marks

4. 15 marks total

5. 20 marks total:
   (a) 10 marks
   (b) 10 marks