NATIONAL EXAMINATIONS MAY 2010

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumption made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks

2. 20 marks

3. (a) 4 marks ; (b) 12 marks ; (c) 4 marks

4. (A) 10 marks ; (B) 10 marks

5. 20 marks

6. (a) 12 marks ; (b) 8 marks

7. (a) 10 marks ; (b) 10 marks

1.Consider the following differential equation

$$(x^{-1}y')' + (1+\lambda)/x^3 = 0 \qquad y(1) = 0;\ y(e) = 0$$

2. Find the Fourier series expansion of the periodic function f(x) of period p=4π.

$$f(x) = \begin{cases} -\pi & -2\pi < x < -\pi \\ x & -\pi < x < \pi \\ \pi & \pi < x < 2\pi \end{cases}$$

3.Consider the following function where a is a positive constant

$$f(x) = \begin{cases} \frac{1}{2a}\exp(x/a) & x < 0 \\ \frac{1}{2a}\exp(-x/a) & x > 0 \end{cases}$$

(a) Compute the area bounded by f(x) and the x-axis.
(b) Find the Fourier transform F(ω) of f(x).

Hint: $$F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx$$

(c) Explain what happens to f(x) and F(ω) when a tends to zero.

4(A).Set up Newton's divided difference formula for the data tabulated below and derive the polynomial of highest possible degree.

x	–4	–2	1	3	4	6
f(x)	363	–13	8	–8	11	403

4(B).Use the forward difference formulas supplied below to find the approximate value of the first, second, third and fourth derivative of the function f(x) tabulated below at x = –2. (Hint: Let x_0 = –2 and h = 1)

x	–4	–3	–2	–1	0	1	2	3	4	5
f(x)	380	144	11	–10	–3	2	–1	6	65	249

$$f'(x_0) \approx \frac{1}{12h}[-25f(x_0)+48f(x_0+h)-36f(x_0+2h)+16f(x_0+3h)-3f(x_0+4h)]$$

$$f''(x_0) \approx \frac{1}{12h^2}[35f(x_0)-104f(x_0+h)+114f(x_0+2h)-56f(x_0+3h)+11f(x_0+4h)]$$

$$f'''(x_0) \approx \frac{1}{2h^3}[-5f(x_0)+18f(x_0+h)-24f(x_0+2h)+14f(x_0+3h)-3f(x_0+4h)]$$

$$f''''(x_0) \approx \frac{1}{h^4}[f(x_0)-4f(x_0+h)+6f(x_0+2h)-4f(x_0+3h)+f(x_0+4h)]$$

5.Use the Romberg algorithm with n = 2 to evaluate the following definite integral.

$$\int_3^5 \frac{30}{\sqrt{9+(x-3)^4}}dx$$

Hint: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(0,0)
R(1,0) R(1,1)
R(2,0) R(2,1) R(2,2)

where

$$R(0,0)=\frac{1}{2}(b-a)[f(a)+f(b)]$$

$$R(n,0)=\frac{1}{2}\ R(n-1,0)+h\sum_{k=1}^{2^{n-1}} f[a+(2k-1)h]$$

$$\text{where } h=\frac{b-a}{2^n}$$

$$R(n,m)=R(n,m-1)+\frac{1}{4^m-1}\ [R(n,m-1)-R(n-1,m-1)]$$

6.(A) The equation $x^x - 7 = 0$ has a root between $\alpha = 2$ and $\beta = 2.5$. (i) Use the method of bisection five times to find a better approximation of this root. (ii) Use Newton's method three times to find a better approximation of this root. Start with the approximation obtained in (a).(Note: Carry seven digits in your calculations)

6(B) Consider the equation $x^3 - 7x^2 + 13x - 7 = 0$. This equation can be written in the form $x = g(x)$ in several ways. Find the root that is close to $x_0 = 4.4$ using the form $x = (7x^2 - 13x + 7)/x^2$ five times. Explain why this form converges to the root.

7.The matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 3 & 1 \\ 9 & -1 & 1 \end{bmatrix}$ can be written as the product of a lower triangular matrix $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and an upper triangular matrix $U = \begin{bmatrix} u_{11} & 0 & 0 \\ u_{21} & u_{22} & 0 \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$ that is A=LU.

(a) Find L and U.

(b) Use the results obtained in (a) to solve the following system of three linear equations:

$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 1 \\ -6x_1 + 3x_2 + x_3 &= 3 \\ 9x_1 - x_2 + x_3 &= 6 \end{aligned}$$