National Exams May 2010

04-CHEM-B1, Transport Phenomena

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an OPEN BOOK EXAM.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 marks. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

Section A: Fluid Mechanics

A1 **[25 marks overall]** A Couette-Hatschek viscometer consists essentially of two vertical, concentric cylinders of length L. The outer cylinder of radius R rotates with angular velocity Ω. The inner cylinder of radius βR ($\beta < 1$) remains stationary.

(a) **[15 marks]** Starting with the appropriate form of the Navier-Stokes equation (see Table A1 on p3 of this exam) and assuming tangential laminar flow of an incompressible fluid in the annular space between the concentric cylinders, and ignoring end effects, show that the velocity distribution is given by:

$$\frac{u_\theta}{r} = \frac{\Omega \cdot \left\{1-\left(\beta^2 R^2\right)/r^2\right\}}{\left(1-\beta^2\right)}$$

(b) **[10 marks]** Furthermore, by selecting the appropriate expression for the shear stress-velocity gradient relationship (see Table A2 on p4 of this exam) show that the velocity can be calculated from measuring the torque (T) according to:

$$\mu = \frac{T \cdot \left(1-\beta^2\right)}{\left(4\pi L \Omega R^2 \beta^2\right)}$$

A2 **[25 marks]** The arrangement of a journal bearing is shown in Fig. A1. It consists of a shaft of diameter $d_1 = 25$ mm, rotating at 10,000 rpm in a journal of length $l = 50$ mm. The shaft is fitted with a collar of external diameter $d_2 = 45$ mm, to withstand axial thrust. The radial clearance between the shaft and the journal (h_1) is 30 μm and the clearance between the thrust collar and the bearing surface (h_2) is 20 μm. If the bearing is supplied with oil having a viscosity of 60 cP, determine the total power to overcome the viscous resistance of the bearing.

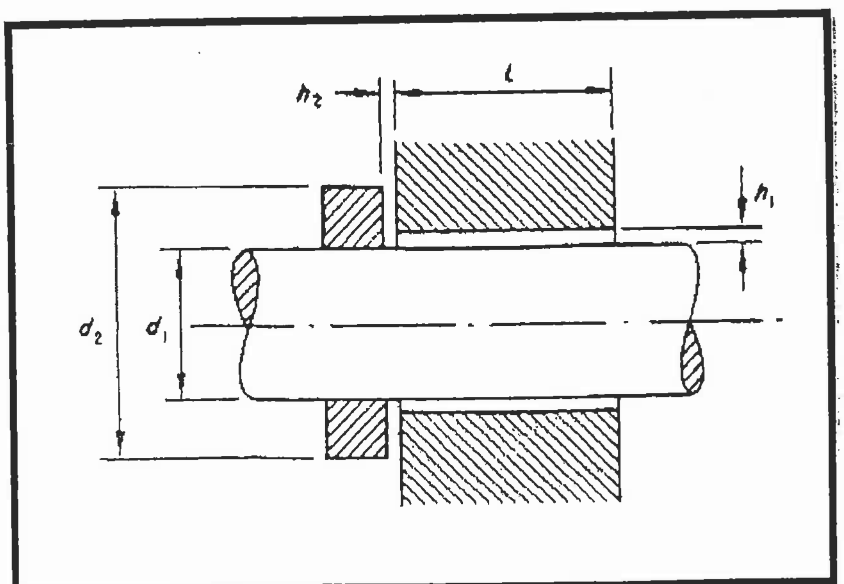

Fig. A1: Journal bearing

Table A1: The Navier-Stokes equations for fluids of constant ρ and μ[1]

Navier–Stokes equation in vector form (rectangular coordinates only)

$$\partial U/\partial t + (U \cdot \nabla)U = -(1/\rho)\nabla p + g + \nu(\nabla^2 U) \tag{5.15}$$

Rectangular coordinates

x component: $$\frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} + U_z \frac{\partial U_x}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) + g_x + \nu\left(\frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2} + \frac{\partial^2 U_x}{\partial z^2}\right) \tag{A}$$

y component: $$\frac{\partial U_y}{\partial t} + U_x \frac{\partial U_y}{\partial x} + U_y \frac{\partial U_y}{\partial y} + U_z \frac{\partial U_y}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) + g_y + \nu\left(\frac{\partial^2 U_y}{\partial x^2} + \frac{\partial^2 U_y}{\partial y^2} + \frac{\partial^2 U_y}{\partial z^2}\right) \tag{B}$$

z component: $$\frac{\partial U_z}{\partial t} + U_x \frac{\partial U_z}{\partial x} + U_y \frac{\partial U_z}{\partial y} + U_z \frac{\partial U_z}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) + g_z + \nu\left(\frac{\partial^2 U_z}{\partial x^2} + \frac{\partial^2 U_z}{\partial y^2} + \frac{\partial^2 U_z}{\partial z^2}\right) \tag{C}$$

Cylindrical coordinates

r component: $$\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_r}{\partial \theta} + U_z \frac{\partial U_r}{\partial z} - \frac{U_\theta^2}{r}$$
$$= -\frac{1}{\rho}\left(\frac{\partial p}{\partial r}\right) + g_r + \nu \frac{\partial^2 U_r}{\partial r^2} + \frac{\nu}{r}\frac{\partial U_r}{\partial r} - \nu\left(\frac{U_r}{r^2}\right) + \frac{\nu}{r^2}\frac{\partial^2 U_r}{\partial \theta^2} - \frac{2\nu}{r^2}\frac{\partial U_\theta}{\partial \theta} + \nu\frac{\partial^2 U_r}{\partial z^2} \tag{D}$$

θ component: $$\frac{\partial U_\theta}{\partial t} + U_r \frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta} + U_z \frac{\partial U_\theta}{\partial z} + \frac{U_r U_\theta}{r}$$
$$= -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + g_\theta + \nu \frac{\partial^2 U_\theta}{\partial r^2} + \frac{\nu}{r}\frac{\partial U_\theta}{\partial r} - \nu\left(\frac{U_\theta}{r^2}\right) + \frac{\nu}{r^2}\frac{\partial^2 U_\theta}{\partial \theta^2} + \frac{2\nu}{r^2}\frac{\partial U_r}{\partial \theta} + \nu\frac{\partial^2 U_\theta}{\partial z^2} \tag{E}$$

z component: $$\frac{\partial U_z}{\partial t} + U_r \frac{\partial U_z}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_z}{\partial \theta} + U_z \frac{\partial U_z}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z + \nu \frac{\partial^2 U_z}{\partial r^2} + \frac{\nu}{r}\frac{\partial U_z}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 U_z}{\partial \theta^2} + \nu\frac{\partial^2 U_z}{\partial z^2} \tag{F}$$

Spherical coordinates

r component: $$\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_r}{\partial \theta} + \left(\frac{U_\phi}{r\sin\theta}\right)\frac{\partial U_r}{\partial \phi} - \frac{U_\theta^2}{r} - \frac{U_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r + \frac{\nu}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial U_r}{\partial r}\right)\right)$$
$$+ \left(\frac{\nu}{r^2\sin\theta}\right)\left(\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial U_r}{\partial \theta}\right)\right) + \left(\frac{\nu}{r^2\sin^2\theta}\right)\left(\frac{\partial^2 U_r}{\partial \phi^2}\right) - \frac{2\nu U_r}{r^2}$$
$$- \frac{2\nu}{r^2}\frac{\partial U_\theta}{\partial \theta} - \frac{2\nu U_\theta}{r^2}\cot\theta - \left(\frac{2\nu}{r^2\sin\theta}\right)\frac{\partial U_\phi}{\partial \phi} \tag{G}$$

θ component: $$\frac{\partial U_\theta}{\partial t} + U_r \frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta} + \left(\frac{U_\phi}{r\sin\theta}\right)\left(\frac{\partial U_\theta}{\partial \phi}\right) + \frac{U_r U_\theta}{r} - \frac{U_\phi^2}{r}\cot\theta$$
$$= -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + g_\theta + \frac{\nu}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial U_\theta}{\partial r}\right)\right) + \left(\frac{\nu}{r^2\sin\theta}\right)\left(\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial U_\theta}{\partial \theta}\right)\right) + \left(\frac{\nu}{r^2\sin^2\theta}\right)\frac{\partial^2 U_\theta}{\partial^2 \phi}$$
$$+ \frac{2\nu}{r^2}\frac{\partial U_r}{\partial \theta} - \left(\frac{\nu U_\theta}{r^2\sin^2\theta}\right) - \left(\frac{(2\nu)\cos\theta}{r^2\sin^2\theta}\right)\frac{\partial U_\phi}{\partial \phi} \tag{H}$$

ϕ component: $$\frac{\partial U_\phi}{\partial t} + U_r \frac{\partial U_\phi}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\phi}{\partial \theta} + \left(\frac{U_\phi}{r\sin\theta}\right)\frac{\partial U_\phi}{\partial \phi} + \frac{U_r U_\phi}{r} + \frac{U_\theta U_\phi}{r}\cot\theta$$
$$= -\left(\frac{1}{\rho r\sin\theta}\right)\frac{\partial p}{\partial \phi} + g_\phi + \frac{\nu}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial U_\phi}{\partial r}\right)\right) + \left(\frac{\nu}{r^2\sin\theta}\right)\left(\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial U_\phi}{\partial \theta}\right)\right)$$
$$+ \left(\frac{\nu}{r^2\sin^2\theta}\right)\frac{\partial^2 U_\phi}{\partial \phi^2} - \left(\frac{\nu U_\phi}{r^2\sin^2\theta}\right) + \left(\frac{2\nu}{r^2\sin\theta}\right)\frac{\partial U_r}{\partial \phi} + \left(\frac{2\nu\cos\theta}{r^2\sin^2\theta}\right)\frac{\partial U_\theta}{\partial \phi} \tag{I}$$

[1] Brodkey, R.S. and Hershey H.C. (1988) *Transport Phenomena – A Unified Approach* Table 5.7 p147.

Table A2: Shear stress-velocity gradient relationships for constant viscosity[2]

Rectangular coordinates

$$\tau_{xx} = -2\mu(\partial U_x/\partial x) + (2\mu/3)(\nabla \cdot U) \quad \text{(A)}$$

$$\tau_{yy} = -2\mu(\partial U_y/\partial y) + (2\mu/3)(\nabla \cdot U) \quad \text{(B)}$$

$$\tau_{zz} = -2\mu(\partial U_z/\partial z) + (2\mu/3)(\nabla \cdot U) \quad \text{(C)}$$

$$\tau_{xy} = \tau_{yx} = -\mu[(\partial U_x/\partial y) + (\partial U_y/\partial x)] \quad \text{(D)}$$

$$\tau_{yz} = \tau_{zy} = -\mu[(\partial U_y/\partial z) + (\partial U_z/\partial y)] \quad \text{(E)}$$

$$\tau_{xz} = \tau_{zx} = -\mu[(\partial U_x/\partial z) + (\partial U_z/\partial x)] \quad \text{(F)}$$

Cylindrical coordinates

$$\tau_{rr} = -2\mu(\partial U_r/\partial r) + (2\mu/3)(\nabla \cdot U) \quad \text{(G)}$$

$$\tau_{\theta\theta} = -2\mu\left[\frac{1}{r}\left(\frac{\partial U_\theta}{\partial \theta}\right) + \frac{U_r}{r}\right] + (2\mu/3)(\nabla \cdot U) \quad \text{(H)}$$

$$\tau_{zz} = -2\mu(\partial U_z/\partial z) + (2\mu/3)(\nabla \cdot U) \quad \text{(I)}$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}(U_\theta/r) + \frac{1}{r}\left(\frac{\partial U_r}{\partial \theta}\right)\right] \quad \text{(J)}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\left(\frac{\partial U_\theta}{\partial z}\right) + \frac{1}{r}\left(\frac{\partial U_z}{\partial \theta}\right)\right] \quad \text{(K)}$$

$$\tau_{rz} = \tau_{zr} = -\mu[(\partial U_r/\partial z) + (\partial U_z/\partial r)] \quad \text{(L)}$$

Spherical coordinates

$$\tau_{rr} = -2\mu(\partial U_r/\partial r) + (2\mu/3)(\nabla \cdot U) \quad \text{(M)}$$

$$\tau_{\theta\theta} = -2\mu\left[\frac{1}{r}\left(\frac{\partial U_\theta}{\partial \theta}\right) + \frac{U_r}{r}\right] + (2\mu/3)(\nabla \cdot U) \quad \text{(N)}$$

$$\tau_{\phi\phi} = -2\mu\left[\frac{1}{r\sin\theta}\frac{\partial U_\phi}{\partial \phi} + \frac{U_r}{r} + (U_\theta/r)(\cot\theta)\right] + (2\mu/3)(\nabla \cdot U) \quad \text{(O)}$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}(U_\theta/r) + \frac{1}{r}\left(\frac{\partial U_r}{\partial \theta}\right)\right] \quad \text{(P)}$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin\theta}{r}\left[\frac{\partial}{\partial \theta}\left(\frac{U_\phi}{\sin\theta}\right)\right] + \frac{1}{r\sin\theta}\frac{\partial U_\theta}{\partial \phi}\right] \quad \text{(Q)}$$

$$\tau_{r\phi} = \tau_{\phi r} = -\mu\left[\frac{1}{r\sin\theta}\frac{\partial U_r}{\partial \phi} + r\frac{\partial}{\partial r}(U_\phi/r)\right] \quad \text{(R)}$$

[2] B&H *ibid* Table 5.2 p137.

Section B: Heat Transfer

B1 [25 marks overall]

(a) **[15 marks]** Consider a hollow sphere of inner radius R_1 and outer radius R_2. If the inner surface is at the higher temperature T_1 and the outer surface is at the lower temperature T_2, starting with the appropriate form of the energy equation (see Table B1 on p6 of this exam) show that the temperature distribution throughout the shell is given by:

$$T = \frac{(T_1 - T_2) \cdot R_2 \cdot (r - R_1)}{r \cdot (R_2 - R_1)}$$

(b) **[10 marks]** Develop an expression for heat loss at the outer surface of the sphere.

B2 [25 marks overall] The following test data has been obtained on a new insulating material.

Test sample	12.77 mm thick by 0.3048 m x 0.3048 m
ΔT across sample	2.77°C
Heat flow	25.32 kJ in a 10-h steady-state test
Sample density	150 kg/m^3
Sample heat capacity	1.255 kJ/kg·K

(a) **[5 marks]** Determine the thermal conductivity of the sample.

(b) **[5 marks]** Determine the thermal diffusivity of the sample.

The new insulating material is formed into blocks 6-in thick and used in a wall sandwiched between 4-in brick (k = 0.72 W/mK) and ½-in drywall (k = 0.17 W/mK). If the heat transfer coefficients on the inside and outside are 8.0 and 12 W/mK calculate the following:

(c) **[5 marks]** the heat flux through the wall when the inner air temperature is 22°C and the outer temperature is 2°C.

(d) **[5 marks]** the temperature of the drywall; and

(e) **[5 marks]** the temperature between the drywall and the insulation.

Table B1: The energy equation[3]

General equation

$$\frac{\partial(\rho c_p T)}{\partial t} + (U \cdot \nabla)(\rho c_p T) = \dot{T}_G + [\nabla \cdot \alpha \nabla(\rho c_p T)] - (\rho c_p T)(\nabla \cdot U) \qquad (5.13)$$

Incompressible media, rectangular coordinates

$$\frac{\partial T}{\partial t} + U_x \frac{\partial T}{\partial x} + U_y \frac{\partial T}{\partial y} + U_z \frac{\partial T}{\partial z} = \frac{\dot{T}_G}{\rho c_p} + \frac{\partial}{\partial x}\left(\alpha \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\alpha \frac{\partial T}{\partial z}\right) \qquad (A)$$

Incompressible media, cylindrical coordinates

$$\frac{\partial T}{\partial t} + U_r \frac{\partial T}{\partial r} + \frac{U_\theta}{r}\frac{\partial T}{\partial \theta} + U_z \frac{\partial T}{\partial z} = \frac{\dot{T}_G}{\rho c_p} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\alpha \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\alpha \frac{\partial T}{\partial z}\right) \qquad (B)$$

Incompressible media, spherical coordinates

$$\frac{\partial T}{\partial t} + U_r \frac{\partial T}{\partial r} + \frac{U_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{U_\phi}{r \sin\theta}\frac{\partial T}{\partial \phi} = \frac{\dot{T}_G}{\rho c_p} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \alpha \frac{\partial T}{\partial r}\right)$$

$$+ \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\alpha \sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(\alpha \frac{\partial T}{\partial \phi}\right) \qquad (C)$$

[3] B&H *ibid* Table 5.6 p143.

Section C: Mass Transfer

C1 [25 marks overall] The mass diffusivity for a gas may be measured in an Arnold diffusion cell as illustrated schematically in Fig. C1. The narrow tube, which is partially filled with liquid A, is maintained at a constant pressure and temperature. Gas B, which flows across the open end of the tube, has negligible solubility in liquid A and is also chemically inert to A. Component A vaporizes and diffuses into the gas phase; the rate of vaporization may be expressed mathematically in terms of the molar flux.

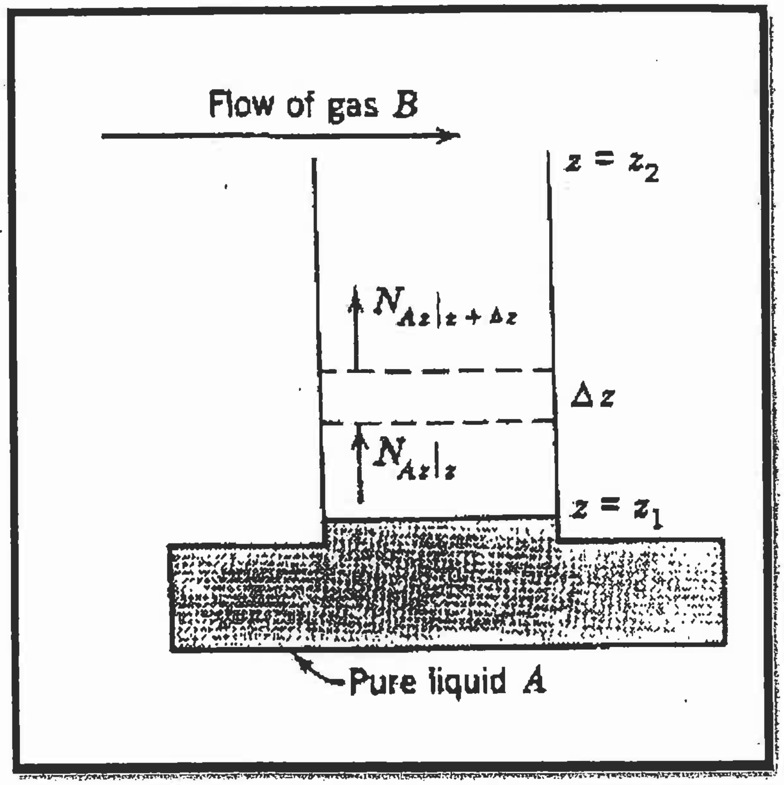

Fig. C1: Arnold diffusion cell

(a) [15 marks] If the concentration of A is C_{A1} and C_{A2} at z_1 and z_2, starting with the appropriate form of the continuity equation for species A (see Table C1 on p 8 of this exam) show that the concentration profile within the tube is given by:

$$C_A = C_{A1} - \frac{(C_{A1} - C_{A2}) \cdot (z - z_1)}{(z_2 - z_1)}$$

(b) [10 marks] Show that the molar flux within the tube is given by:

$$N_A = D_{AB} \cdot \frac{(C_{A2} - C_{A1})}{(z_2 - z_1)}$$

Table C1: The continuity equation for species A[4]

General equation

$$\frac{\partial C_A}{\partial t} + (U \cdot \nabla)C_A = \dot{C}_{A,G} + (\nabla \cdot D\nabla C_A) - (C_A)(\nabla \cdot U) \qquad (5.8)$$

Incompressible media, rectangular coordinates

$$\frac{\partial C_A}{\partial t} + U_x\frac{\partial C_A}{\partial x} + U_y\frac{\partial C_A}{\partial y} + U_z\frac{\partial C_A}{\partial z} = \dot{C}_{A,G} + \frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) \qquad \text{(A)}$$

Incompressible media, cylindrical coordinates

$$\frac{\partial C_A}{\partial t} + U_r\frac{\partial C_A}{\partial r} + \frac{U_\theta}{r}\frac{\partial C_A}{\partial \theta} + U_z\frac{\partial C_A}{\partial z} = \dot{C}_{A,G} + \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) \qquad \text{(B)}$$

Incompressible media, spherical coordinates

$$\frac{\partial C_A}{\partial t} + U_r\frac{\partial C_A}{\partial r} + \frac{U_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{U_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi} = \dot{C}_{A,G} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2D\frac{\partial C_A}{\partial r}\right)$$
$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right) \qquad \text{(C)}$$

C2 [25 marks overall]. Two engineers are engaged in a heated discussion about a required calculation for a given system that involves a large amount of water in parallel flow to a benzoic acid plate. Since benzoic acid is soluble in water (solubility of 0.02595 kg-mol/m^3), the molar flux (kg-mol/s) must be determined.

Engineer A argues that $(k_c)_{av}\frac{P_{BM}}{uP}(Sc)^{2/3} = j_D = 0.037(\mathrm{Re})^{-0.2}$ can be used by simply letting (P_{BM}/P) be unity because the system involves a liquid.

Engineer B counters that what should be used is the relationship $j_D = 0.99(\mathrm{Re})^{-1/2}$ developed for flowing liquids.

The plate is 0.3 m long and the water velocity over the plate is 0.05 m/s. One can use the properties of water (viscosity = 8.71x10^{-4} kg/ms and density = 996 kg/m^3) for calculation of the dimensionless groups since the flowing liquid will have a dilute concentration of benzoic acid. The diffusivity of benzoic acid in water is 1.24x10^{-9} m^2/s. Estimate the percentage difference in the molar flux determined by each engineer.

[4] B&H *ibid* Table 5.4 p142.