Association of Professional Engineers of Ontario

National Examinations May 2010

07-Elec-A3 Signals and Communications

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.
2. This is a Closed Book exam – no aids other than a approved Casio or Sharp calculator
3. There are six questions in total, and any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. A DSB-SC (double sideband suppressed carrier) AM signal is given by

$$s(t) = m(t)\cos(2\pi f_c t),$$

where $m(t) = \cos(2\pi f_m t)$, with $f_m \ll f_c$.

(a) If $s(t)$ is passed through an ideal envelope detector, sketch the output of the detector.

(b) If $s(t)$ is passed through a coherent demodulator that suffers from a phase offset of θ radians, i.e. it generates the sinusoid $\cos(2\pi f_c t + \theta)$ instead of the correct $\cos(2\pi f_c t)$, what is the output of the demodulator?

(c) Let $q(t)$ be a periodic signal with fundamental frequency $f_c/2$, and Fourier coefficients q_k, i.e.

$$q_k = \frac{f_c}{2}\int_0^{2/f_c} q(t)e^{-j\pi k f_c t}dt,$$

with $q_2 = 0.5$. How do we process $m(t)q(t)$ to obtain $s(t)$?

2. Let $m(t)$ be the signal with the spectrum

$$M(f) = \begin{cases} -\frac{|f|}{1000} + 1, & 0 \le |f| \le 1000 \\ 0 & \text{elsewhere} \end{cases}$$

with f in Hertz.

(a) What is the minimum sampling rate that can be used on $m(t)$ to avoid aliasing?

(b) If we sample at the minimum rate found in part (a) and use an 8-bit quantizer, find the bit rate at the output of a pulse-code modulation (PCM) encoder.

(c) If the output of the PCM encoder of part (b) is to be transmitted using quaternary baseband pulse amplitude modulation (PAM), find the minimum bandwidth required for zero inter-symbol interference (ISI).

3. Let $a[n]$ be the sequence of symbols transmitted through a channel, modeled as a linear time-invariant (LTI) system with impulse response $h[n]$. The channel output is

$$y[n] = 0.5a[n] + 0.1a[n-1].$$

   (a) Find the impulse response of the channel, $h[n]$.

   (b) The symbols are independently drawn from a quaternary phase shift keying (QPSK) constellation with symbol energy $E_s = 1$. Find the signal to interference ratio (SIR) in the channel output $y[n]$, assuming the receiver uses $y[n]$ to detect $a[n]$, i.e. $a[n-1]$ is inter-symbol interference (ISI).

   (c) A linear zero-forcing equalizer filters $y[n]$ with a filter having the impulse response $g[n]$ satisfying $g[n] * h[n] = \delta[n]$ where $*$ is the convolution operator. Write down the difference equation that describes the zero-forcing equalizer for this channel.

4. Consider the 8-ary passband modulation format that chooses signals with equal probability from the set

$$S = \left\{ q\sqrt{\frac{2}{T}} \cos\left[\omega_c t + \frac{\pi}{2}p + \frac{\pi}{4}(q-1)\right] : \ q = 1, 2; p = 0, 1, 2, 3 \right\}.$$

   (a) Sketch the signal constellation in 2-D signal space.

   (b) Find the average energy transmitted per bit.

   (c) Now consider an 8-PSK modulation format. Express the 8-PSK signal constellation in the same manner as above, for the same average bit energy.

   (d) Does the 8-PSK or the original constellation have better noise tolerance? Explain your answer briefly.

5. Consider the modulating (message) signal $m(t)$ given by

$$m(t) = \begin{cases} t & 0 < t \leq 5 \\ -t + 10 & 5 < t \leq 10 \end{cases}$$

with t measured in milliseconds, and $m(t)$ is periodic with a period of 10 milliseconds. The message $m(t)$ is to be phase modulated (PM), with a transmitted signal given by

$$\phi_{PM}(t) = A \cos\left[2\pi f_c t + k_p m(t)\right], .$$

where $k_p = 2\pi \times 10^5$ and $f_c = 100$ MHz.

(a) Using Carson's Rule, estimate the bandwidth occupied by $\phi_{PM}(t)$.

(b) If we assume the bandwidth of $m(t)$ to be the third harmonic frequency, find the Nyquist sampling rate for $m(t)$.

(c) We sample $m(t)$ at its Nyquist rate, and quantize the samples using an 8-bit quantizer. What is the minimum bandwidth required for transmission of the binary pulse amplitude modulated (PAM) signal?

6. Let the Fourier transform of $x(t)$ be $X(f)$.

(a) Find the Fourier transform of $x(t) * x^*(-t)$, where $*$ denotes convolution.

(b) If $x(t)$ is real-valued, show that $X(f) = X^*(-f)$.

(c) If $x(t)$ is a real-valued passband signal, i.e. it has all its energy in the frequency band $f_c - B < f < f_c + B$, sketch the spectrum of

$$\tilde{x}(t) = \mathcal{F}^{-1}\left[X(f + f_c)u(f + f_c)\right]$$

where $u(f) = 1$ if and only if $f \geq 0$. Is $\tilde{x}(t)$ real-valued in general?

(d) Draw a block diagram for a "down-converter" that transforms $x(t)$ into $\tilde{x}(t)$. Assume that a filter with transfer function $u(f)$ is available.