Professional Engineers of Ontario

Annual Examinations – May 2010

07-Elec-B3
Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. A PEO-approved non-programmable calculator is permitted.
3. There are **6 questions** on this exam. **Any 5 questions constitute a complete paper.** Only the first 5 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(**20 marks**) 1. Suppose you are asked to design a system for transmitting data over a wireless link, with a bandwidth of 1 MHz, using a 16QAM constellation.

(6 marks) a. Briefly explain how a signal is transmitted using the rectangular 16QAM constellation.

(7 marks) b. Using 16QAM symbols and raised-cosine pulses with an excess bandwidth of 0.5, what is the data rate (in bits per second) at which information can be transmitted over this link?

(5 marks) c. Supposing the signal-to-noise ratio (SNR) of the link is 3 dB, under the Shannon limit, is it possible to reliably communicate at the data rate found in part a? What is the maximum data rate at which information can be reliably transmitted?

(2 marks) d. For the link in part b, a vendor approaches you and tells you about a new technology which, according to the vendor's claim, allows data transmission at a rate faster than the rate you calculated. Are the vendor's claims reasonable? Why or why not?

(**20 marks**) 2. This question concerns source coding.

(10 marks) a. Suppose you have a source with five letters: A, B, C, D, and E; with probabilities given by Pr(A)=1/2, Pr(B)=1/4, Pr(C)=1/8, Pr(D)=Pr(E)=1/16. Give the Huffman code for this source, showing all your work.

(4 marks) b. Calculate the entropy of the source in part a.

(4 marks) c. Calculate the average length of the Huffman code you derived in part a.

(2 marks) d. Considering your answers to all of the above questions, does there exist any uniquely decodable source code with average length shorter than the Huffman code you derived?

(**20 marks**) 3. This question concerns sampling and conversion.

(4 marks) a. The power spectrum for the human voice spans the range from 0 Hz to 4 kHz. What is the sampling frequency that is required to exactly reconstruct a human voice signal?

(8 marks) b. Briefly explain pulse code modulation (PCM). If PCM is used to encode your signal from part a with 8 bits per sample, what is the required data rate to represent the signal?

(8 marks) c. Briefly explain how delta modulation is used in analog-to-digital and digital-to-analog conversion.

(20 marks) 4. This question concerns the use of spread spectrum modulation.

(8 marks) a. Explain the operation of frequency hopping spread spectrum, including signal modulation and detection. In what sense is this technique "spread spectrum"?

(8 marks) b. Explain the operation of direct sequence spread spectrum, including signal modulation and detection. In what sense is this technique "spread spectrum"?

(4 marks) c. Briefly explain why power control is needed in a direct sequence spread spectrum system.

(20 marks) 5. This question concerns signal modulation and detection.

(5 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_0(t) = \begin{cases} \sin(2\pi t/T), & 0 \le t \le T; \\ 0, & t < 0, t > T; \end{cases}$$

and $s_1(t) = -s_0(t)$. Sketch the two signals, and sketch the impulse response of the matched filter (assuming the filter is matched to $s_0(t)$, and assuming the filter output is sampled at time T).

(8 marks) b. Assume these signals are observed through an additive white Gaussian noise process $N(t)$ with power spectral density $S_N(f) = N_0/2$ (i.e., constant with respect to f). Let $Z(t)$ be the output of the matched filter, where the input is the signal plus noise. Give the mean and variance of the random variable $Z(T)$ given that 0 was sent, and given that 1 was sent.

(7 marks) c. Give the optimal decision rule for the system assuming that 0 and 1 are equiprobable, and describe how to calculate the probability of error (but don't do the calculation).

(20 marks) 6. This question concerns error-control coding.

(6 marks) a. Consider a binary error-correcting code with a minimum Hamming distance of 7. At most how many errors can be corrected, and at most how many errors can be detected? Explain.

(6 marks) b. Consider a binary Hamming code with the following parity check matrix:

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the generator matrix of this Hamming code.

(8 marks) c. Given the parity check matrix of a Hamming code (such as the one in part b), explain how to correct a single error.