# National Exams May 2010
# 07-Mec-A2, Kinematics and Dynamics of Machines
3 Hours in Duration

**Notes:**

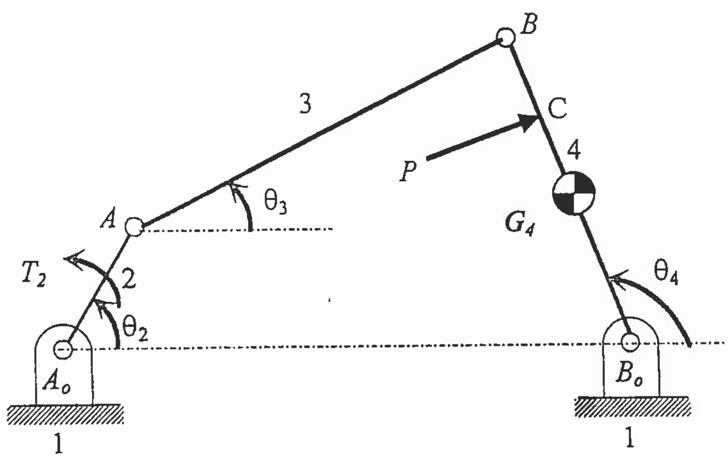
1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK exam. Any Sharp or Casio approved calculators are permitted.
3. Four questions in the following combinations constitute a complete exam paper: two from part A and two from part B.
4. All questions are of equal value.

Part A

1. [25] For the four bar mechanism shown below, determine analytically at the instant
(i) the angular velocity and angular acceleration of the follower
(ii) the linear accelerations of the mass center of the follower, and
(iii) torque, $T_2$, required to drive the mechanisms and overcome the external load and the inertia
(iv) forces provided by the frame at $A_o$ and $B_o$.

Note: The masses of the input link and the coupler are negligible.

Not to scale.




Geometry and material property parameters:
$A_oB_o$ = 228 mm
$A_oA$ = 57 mm
$AB$ =165 mm
$B_oB$ = 129 mm
$BC$ =33 mm
$BG_4$ =64.5 mm
$m_4$=0.5 kg
$I_{G4}$=0.05 kg•m$^2$

Configuration and operational parameters:
$\theta_2$=58.5°, $\theta_3$=25.4°, $\theta_4$=112.4°
$\omega_2$=40 rad/s(ccw), $\alpha_2$ = 0 rad/s$^2$
$P$ = 250 N (normal to $BB_o$)

2. [25] For a planetary gear train shown below, gear 3 is fixed; gear 5 is attached to the input shaft. If the numbers of teeth are $N_1 = 16$, $N_3 = 80$, $N_4 = 23$, $N_5 = 59$, determine
   (a) the speed and direction of rotation of the output shaft, and
   (b) relative rotational speed of planet carrier (arm) with respect to gear 4.

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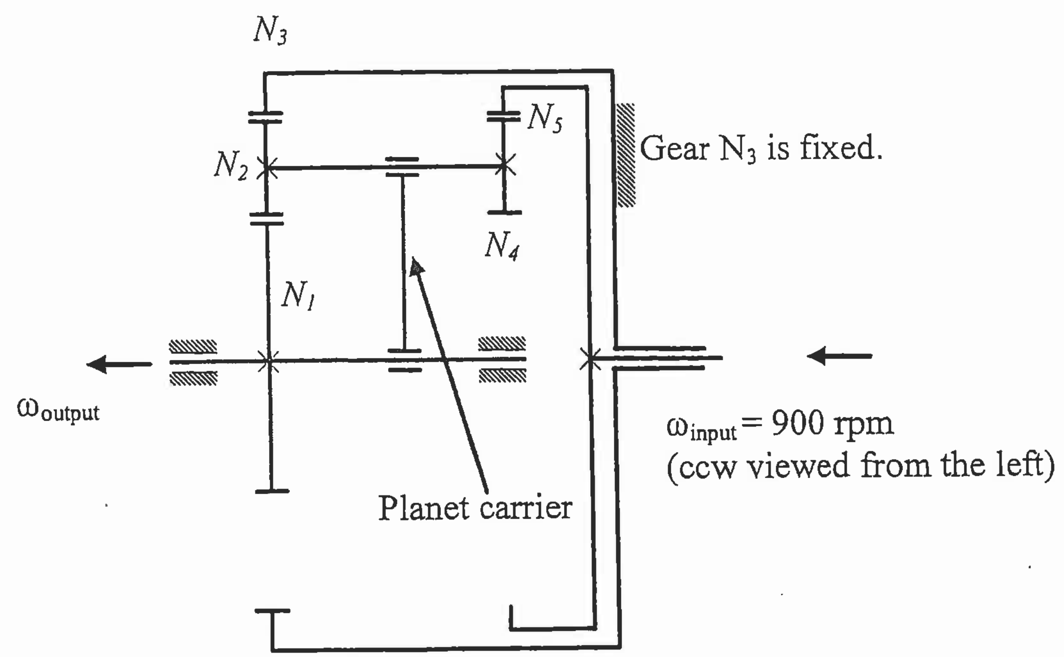


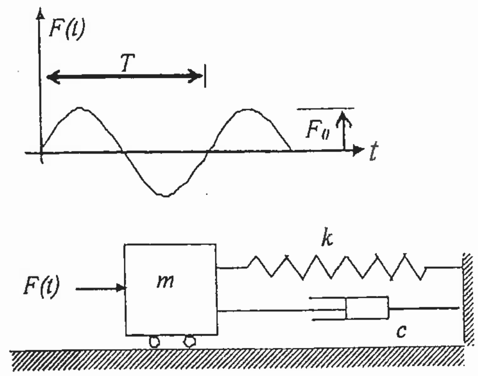

3. [25] A cam, rotating at a constant angular velocity of 300 rad/s, is used to produce a 10 mm follower lift with the following specifications:
   Rise: from 0 to 10 mm during [0, 75°],
   Dwell: at the 10 mm lift during [75°, 255°], and
   Fall: from 10 mm back to 0 mm during [255°, 360°]

Design the displacement profile of the cam during the rise. Since the cam is operated at a high speed, you are required to ensure that (i) the profile satisfies the law of cam design and (ii) both the jerk and the maximum acceleration should be kept as small as possible.

You must present the equations of displacement, velocity, and acceleration and jerk for the rise period, and sketch the rise profile for $s$, $v$, $a$, and $j$, and compute the maximum acceleration and the maximum jerk for your cam.

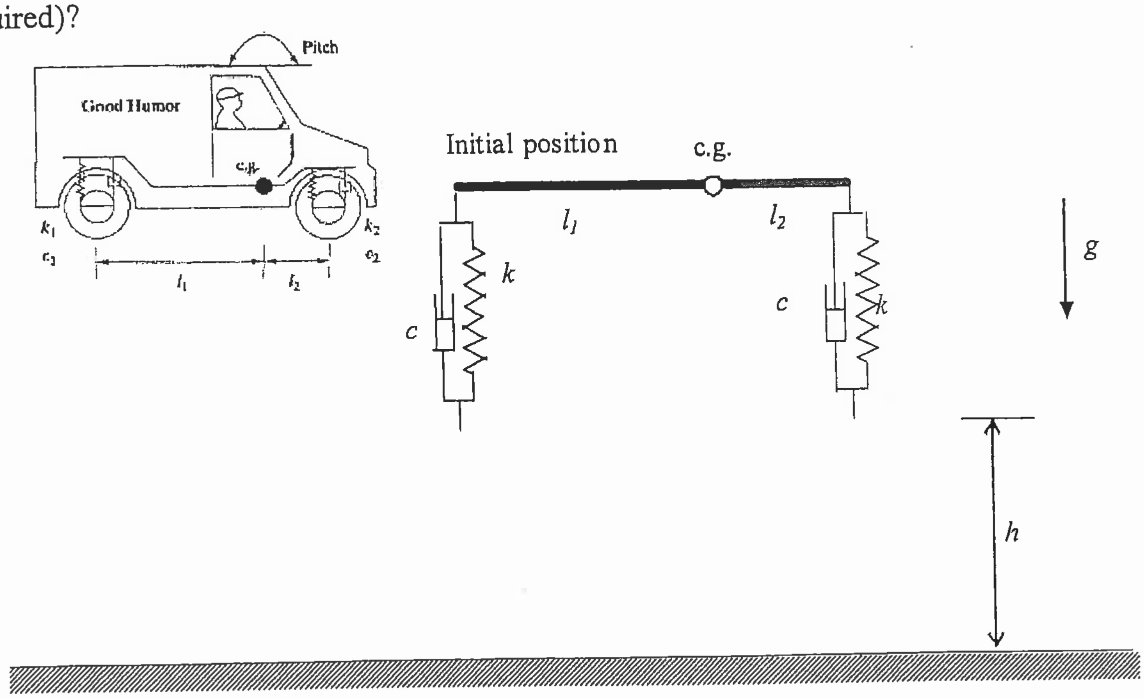
## Part B

4. [25] A single DOF vibration system is subjected to a harmonic excitation as shown below. Determine the steady state response of the system and the peak force transmitted to the vertical support through the spring and the dashpot. Values of parameters are: $m$ = 10 kg, $c$ = 20 Ns/m, $k$ = 1000 N/m, T = 1.2 second, $F_0$ = 10 N.

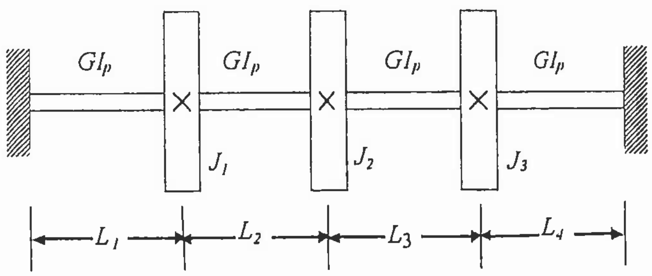


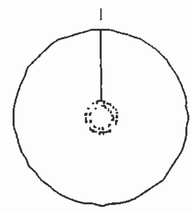
5. [25] To determine the actual vibrational characteristics of a 4-wheel vehicle for bouncing and pitch motions, a drop test is introduced. For simplicity, the vehicle is modelled as a rigid bar constrained by the front and rear suspension (identical springs and dashpots). Initially, the bar is suspended at $h$ distance above the ground; the rigid bar is horizontal. After the tires impact the ground, it is assumed that they remain in contact with the ground for the ensuing vibration due to the large gravitational force (or grip).

(a) Choose an appropriate set of generalized coordinate and establish the equations of motion for the two DOF system.
(b) Determine the two natural frequencies, two modal vectors, two normalized modal vectors of the undamped vibration system.
(c) Determine the ensuing motion of the vehicle after impact with the ground and the maximum dynamic forces acting on the ground (ignore damping)
(d) Would the vehicle leave the ground (i.e., the largest dynamic forces are larger than the static forces at the contact points)? (ignore damping)
In all of your calculations, use $c_1 = c_2 = 0$ Ns/m, $m = 1000$ kg, $I_G = 40$ kg m$^2$, $l_1 = 1.5$ m, $l_2 = 0.75$ m, $k_1 = k_2 = k = 200\,000$ N/m, $h = 2$ $m$, g = 9.81 m/s$^2$.
(e) Explain briefly and clearly how you would consider the effect of damping (no calculations are required)?



6. [25] A shafting system consists of a massless steel (circular) shaft and three disks (gears). We are concerned with the torsional vibration behaviors of the rotor system. Therefore, only the torsional strain energy and the rotational kinetic energies of the three disks about the shaft axis are considered. Lateral bending is ignored in this problem. Choose a proper set of coordinates and establish the equations of motion for torsional vibration of the multiple DOF system. Find one of the natural frequencies and its corresponding mode shape (vector). Values of parameters are $d$ =48 mm, $L_1 = L_2 = L_3 = L_4 = 100$ mm; G = 70 GPa, $J_1 = J_2 = J_3 = 0.01$ kg m$^2$.



Side View