National Exams May 2010

07-Mec-A3, SYSTEM ANALYSIS AND CONTROLS

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1. a) The characteristic equation for a feedback control system is:

$$(s+2)\ (s^2+4s+8)\ +\ K = 0$$

Determine the range of values of K for which the system is stable.

b) Use the Lapalce transform method to solve the following differential equations.

- $\dfrac{dy}{dt} + y = 2\sin t$
- $\dfrac{dy}{dt} + y = 2\cos t$

All the initial conditions are zero.

2. a) Construct the asymptotic Bode plots for the frequency response function:

$$GH(jw) = \frac{1 + jw/2 - (jw/2)^2}{jw(1+jw/0.5)(1+jw/4)}$$

b) Find the gain margin and the phase margin.

3. Draw the root locus for the following open-loop transfer function.

$$GH(s) = \frac{K}{(s+1)\ (s^2+s+1)}$$

Determine the range of the gain for which the system is stable.

4. a) A feedback control system has a characteristic equation:

$$s^3 + (4+K)\,s^2 + 6s + 16 + 8K = 0$$

The parameter K must be positive. What is the maximum value K can assume before the system becomes unstable? When K is equal to the maximum value, the system oscillates. Determine the frequency of oscillation.

b) The dynamics of a system are described by the differential equation:

$$y(t) = \frac{10\ (2D+1)}{(D+2)\ (D+5)}\ f(t)$$

where $D = \dfrac{d}{dt}$

Use the Laplace transform method to determine the response y(t) when all initial conditions are zero and the forcing function f(t) is a unit step function.

5. a) Using the Routh-Hurwitz criterion, investigate the stability of the following characteristic equation:

$$s^5 + s^4 + 2s^3 + s^2 + s + K = 0$$

b) What is the unit step response of a system whose transfer function is given by

$$P(s) = \frac{(s + 2)}{(s + 0.5)(s + 4)}$$

c) A system is designed to give satisfactory performance when a particular amplifier gain K has the value 2. Determine how much this gain can vary before the system becomes unstable if the characteristic equation is:

$$s^3 + (4 + K)s^2 + 65 + 16 + 8K = 0$$

6. The block diagram of a control system is shown in Fig. 1.

a) When $k_i = 10$, r(t) is a unit step function and d(t) = 0. Obtain the value of the steady-state error.

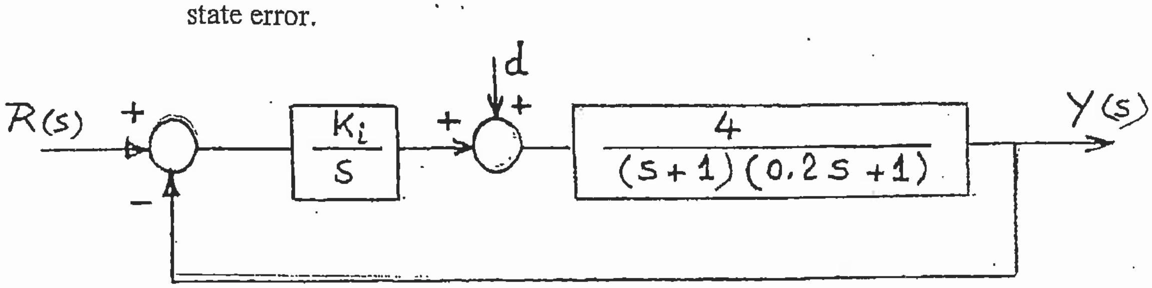

Fig. 1 Block diagram of a control system

b) When the input is a ramp such that r(t) = t, d = 0, it is desired to limit the steady-state error to a value equal to or less than 0.2. Obtain the value of k_i and determine if this requirement is consistent with the requirement of stability.

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{2}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$

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Semi-Logarithmic
3 Cycles x 10 to the inch