PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – May 2010

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. **If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) that he/she has had made with the answer.**

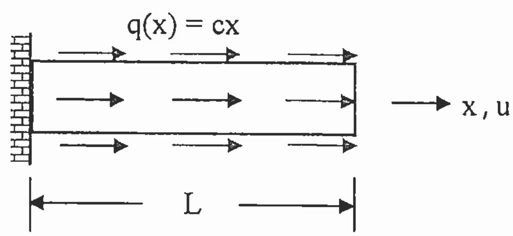
2. **The examination paper is open book and so candidates are permitted to make use of any textbooks, references or notes that they wish to use.**

3. **Any non-communicating calculator is permitted. A calculator that can handle small matrices will speed the solving of the problems. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**

4. **Candidates are required to attempt five questions. Solve all problems using finite element method.**

5. All questions carry the same value. Indicate which five questions are to be marked on the cover of the first examination workbook.

Question 1. [20 marks] A cantilevered bar is loaded by a linearly varying distributed load $q(x) = cx$ as shown in the figure - note that c is a constant. The cross-sectional area and length of the bar are denoted by A and L, respectively, and it is made of a material with Young's modulus of elasticity E. The system governing equation can be written as

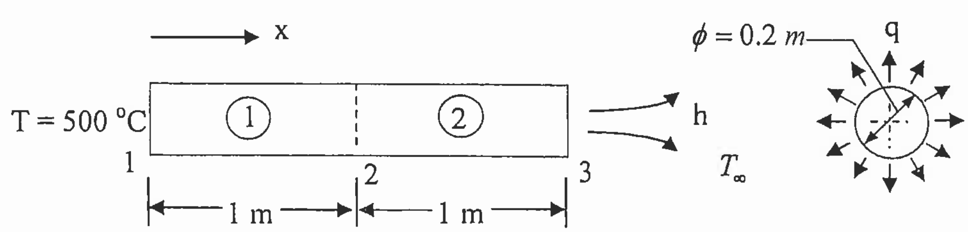


$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\textit{subject to}: \; u(x=0) = 0 \quad \textit{and} \quad EA\frac{du(x)}{dx}\bigg|_{x=L} = 0$$

Use the Bubnov-Galerkin method (i.e., Galerkin method) to determine an approximate cubic polynomial solution.

Question 2. [20 marks] The figure below depicts a 2 m long rod with diameter $\phi = 0.2$ m. The rod is discretized into two elements of equal lengths numbered 1 and 2 as shown.

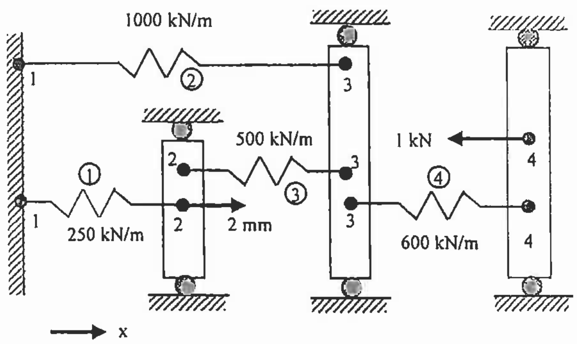


The thermal conductivity in the x-direction of the first and second element is $K_{xx}^{(1)} = 200$ W/m.°C and $K_{xx}^{(2)} = 400$ W/m. °C, respectively. A uniform internal heat source $Q^{(1)} = 400$ W/m^3 is situated in the first element and that in second element is $Q^{(2)} = 500$ W/m^3. Further, a uniform outgoing heat flux $q_{out} = 10$ W/m^2 acts over the whole cylindrical surface of the rod. The temperature at the left-hand end of the rod is maintained constant at 500 °C. Heat convection arises **only** at right-hand end of the rod with convection heat flux $h = 20$ W/m^2. °C and the downstream temperature $T_\infty = 100$ °C. Determine

a) the temperature at nodes 2 and 3,

b) the heat flux over the left portion (i.e., element 1),
c) the heat flow rate and the direction at node 1.

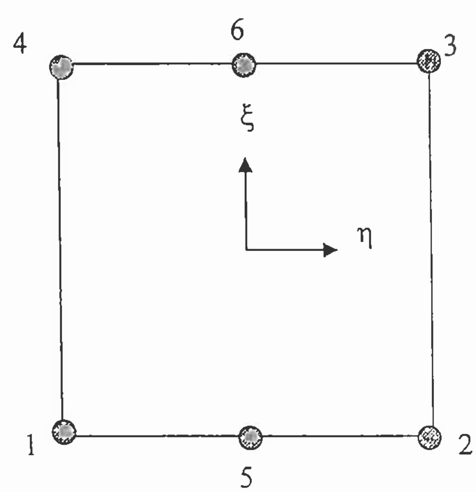
Question 3. [20 marks]



For the spring assemblage shown in the above figure, determine

a) the global system equation,
b) the reduced global system equation,
c) the nodal displacements and reaction forces,
d) and the forces in element #3 only.

Question 4. [20 marks]
A six-noded transition element is shown in the figure below.

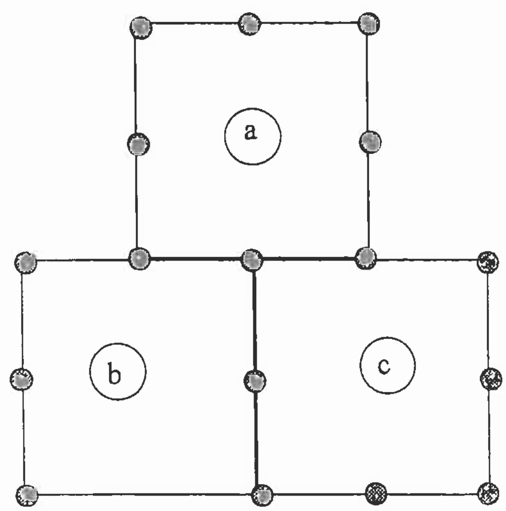


(a) Determine the shape functions.

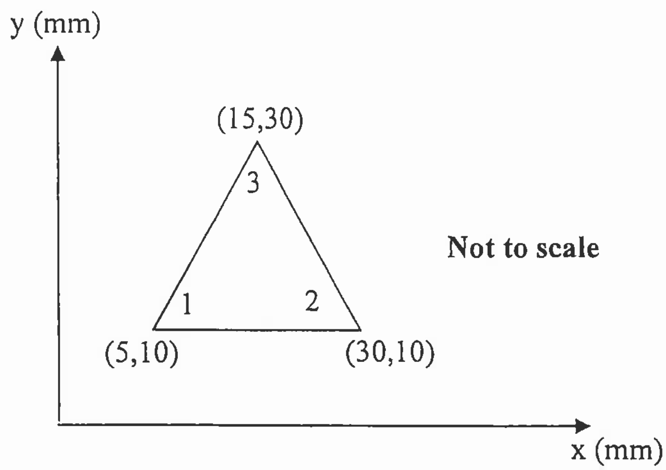
(b) Explain whether the shape function $N_4(\xi,\eta)$ satisfies C^0 interelement continuity along the sides that contain node 4.

(c) Explain whether this element can connect to a nine-node biquadractic quadrilateral element along side 1-5-2 while satisfying compatibility?

(d) Explain the problem in the finite element mesh below involving 8-node serendipity elements, namely a, b, and c.



Question 5. [20 marks]



The nodal displacements for the plane strain element shown in the figure above are:

$u_1 = 0.005$ mm and $v_1 = 0.002$ mm; $u_2 = v_2 = 0.0$ mm; $u_3 = 0.005$ mm and $v_3 = 0.0$ mm

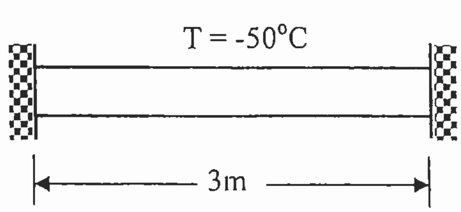
The plate thickness $t = 1$ mm, and it is made from a material with Young's modulus $E = 210$ MPa and Poisson's ratio $\nu = 0.3$.

(a) Determine the element stresses σ_x, σ_y, and τ_{xy}.

(b) Determine the principal stresses σ_1 and σ_2 and principal angle θ_p.

Question 6. [20 marks]

A one-dimensional steel bar fixed at each end is shown in the figure. The properties of the bar are: Young's modulus $E = 210$ GPA, cross-sectional area $A = 1 \times 10^{-2}$ m^2, and coefficient of thermal expansion $\alpha = 12 \times 10^{-6}$ (mm/mm)/°C. If the bar is subjected to a uniform temperature drop of $T = 50$°C as shown, determine the reactions at the fixed ends and the stress in the bar.



Question 7. [20 marks]

(a) [3 marks] Briefly explain in a sentence or two the difference between basis function and shape function.

(b) [3 marks] Briefly explain in a sentence or two why finite element solutions improve with increasing number of elements.

(c) [4 marks] Briefly explain in a sentence or two why a discretization implemented using Bubnov-Galerkin method (i.e., Galerkin method) is identical to that obtained using Ritz method.

(d) [10 marks] An analyst notices a very slow convergence during a finite element analysis of a Timoshenko beam. On closer examination of the field variables, it is noticed that the transverse displacement was interpolated with a quadratic function and rotation by a linear function. What phenomenon is the analyst experiencing and how can the problem be resolved?