National Exams May 2010

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. Answer any **5** questions.
4. Weighting: All questions have an equal weighting (20%)

Question 1: A wind tunnel consists of a straight test-section and an expanding section (the diffuser) as shown in the Fig. 1. During a test, a stationary normal shock forms in the test section. Upstream of the shock, air (γ = 1.4, R = 287 J/kg-K) is moving supersonically at a Mach number of 2 and the pressure and temperature are measured as P = 42kPa and T= -40°C, respectively. At the diffuser outlet, the static pressure is measured to be 200kPa. If the diffuser outlet has a cross-sectional area of 1.5m^2, and neglecting frictional losses along the walls, determine:

(a) The Mach number and temperature of the flow at the diffuser outlet.
(b) The mass flow rate of air through the test section.
(c) The test section area?

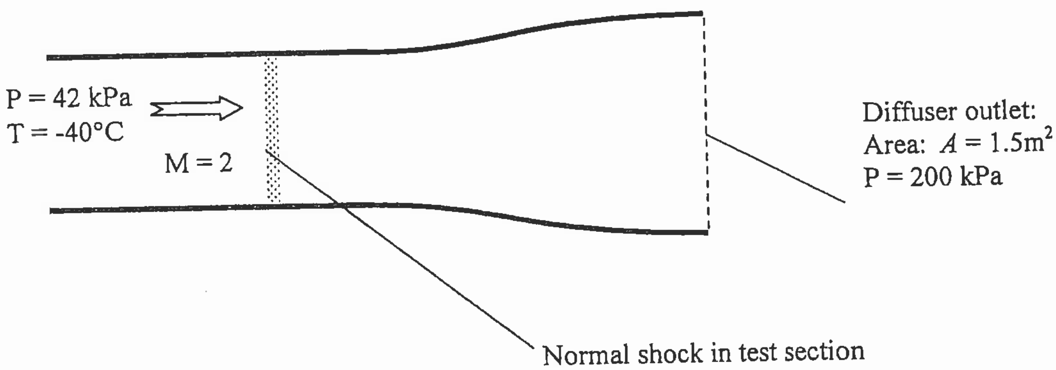


Figure 1: Schematic representation of supersonic wind tunnel with normal shock in the test section.

Question 2: A small lawn sprinkler is shown in Fig. 2. The sprinkler operates at a gauge pressure of 140 kPa. The total flow rate of water through the sprinkler is 4 liters per minute. The exit hole is 1.6mm in diameter. Each leg is symmetric and discharges water in a direction inclined 30° above the horizontal. The sprinkler rotates about a vertical axis. Friction on the bearing causes a torque of 0.18 N-m (opposing rotation). Determine the steady state speed of rotation of the sprinkler. What is the torque needed to hold the sprinkler stationary?

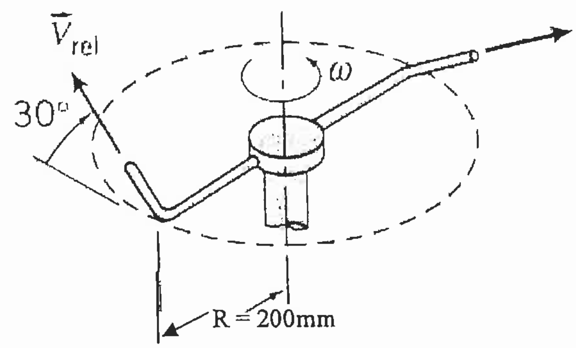


Figure 2: Sprinkler schematic.

Question 3: Consider the system shown in Fig. 3 below. A pump drives the flow at a rate 0.04 m^3/s from the lower to the upper reservoir through the system which consists of a 10-cm inner diameter wrought iron pipe (effective roughness, 0.05mm). The pump is driven by a 17.2 kW generator and has a hydrualic efficiency of 75%. Find the difference in elevation H of the two reservoirs. For the calculation of the major losses, you may assume that the pipe is continuous (i.e. the pump does not affect the major losses). The valve is a globe valve fully open. The entrance loss coefficient is given as 0.5. For water, use ρ = 1000 kg/m^3 and μ = 0.001 kg/m-s. The reservoirs may be considered large. The pipe is of constant diameter 10cm (all segments).

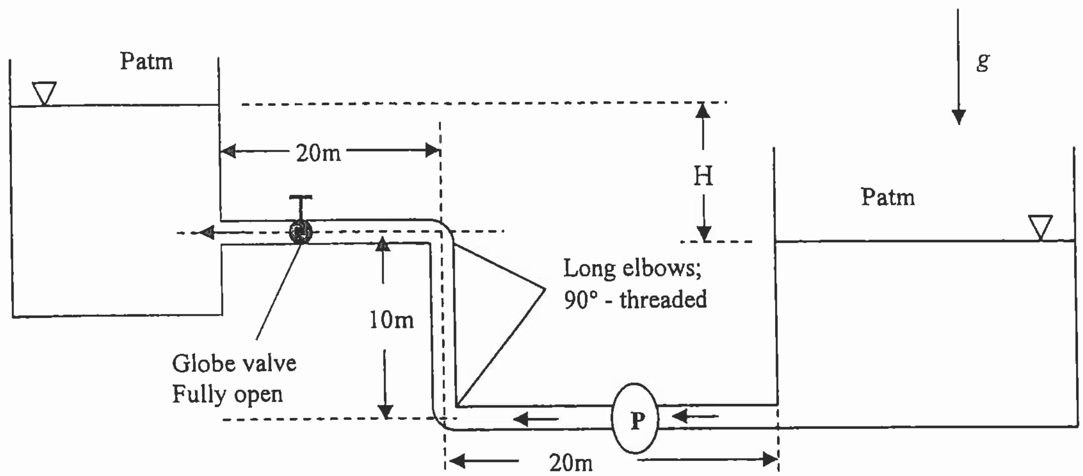


Figure 3: Pipe network connecting two large reservoirs at different elevations.

Question 4: Determine the flow rate of water and the forces acting on the flange A-A if water discharges at an angle of $\alpha = 45°$ as shown in the figure below. A U-tube manometer, with one end open and filled with mercury ($\rho = 13{,}600 kg/m^3$), is attached at the level of the flange and reads height of h = 2cm. The pipe has a uniform internal diameter of 2cm. The flow issues through an orifice plate (mounted at the exit of the pipe) with a hole of diameter 1.414cm. The network has a loss coefficient of K=1 based on the internal flow speed, U_1. The volume of water in the pipe between the exit and the flange A-A can be taken as: $(H+L)\pi D^2/4$, where H = 0.816m, L = 0.5m and D = 0.02m.

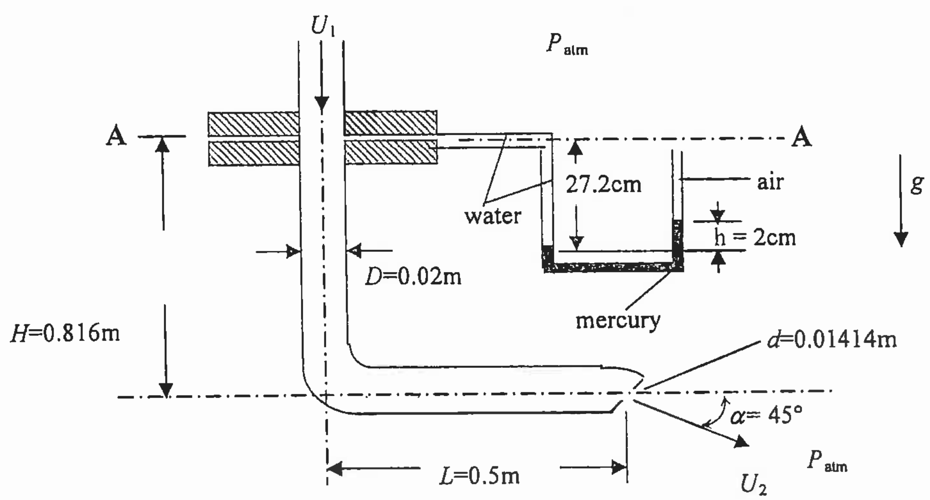


Figure 4: Schematic of pipe system.

Question 5: A stream of water impacts on a block sliding along the ground as shown in the figure below. The coefficient of sliding friction is given as c_μ=0.05. The mass of the cart is M = 100kg. It may be assumed that the fluid losses are negligible and that the gravitational potential on the water stream is negligible compared to the kinetic energy.

The jet issues at a flow rate of Q = 0.05m^3/s and has cross-sectional dimensions of h = 5cm and b = 20cm (b is into the page).

a) Determine the terminal speed of the block.
b) What is the initial acceleration of the block (*i.e.* when block initially at rest)?

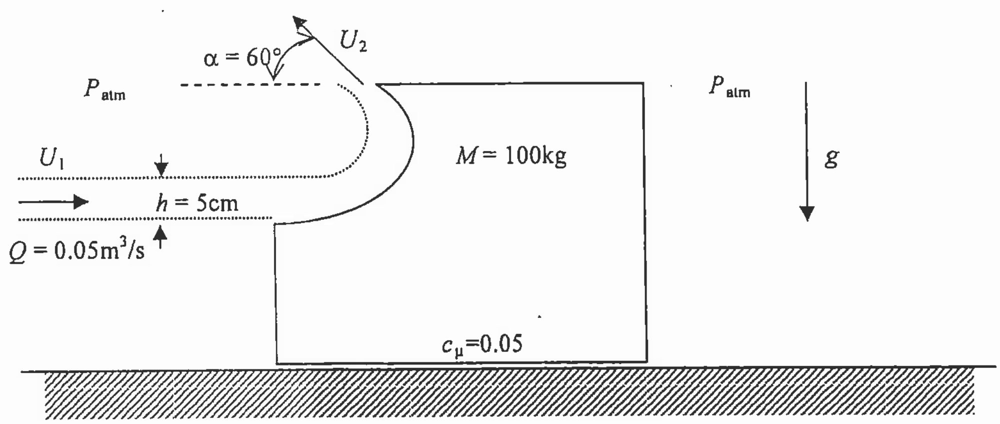


Figure 5: Water jet impacting on a sliding block.

Question 6: Consider the flow of an incompressible, Newtonian liquid (of density ρ and dynamic viscosity μ) in a thin vertical enclosure of width h with one wall moving at a constant rate U_p, as shown schematically below. The enclosure is long ($L/h >> 1$) such that the flow may be assumed fully developed. You can also assume that the span (b into the page) is large ($b/h >> 1$). It is further known that the liquid wets the solid (non-porous) walls and that the flow is not changing in time.

a) State the simplifying assumptions.
b) What are the boundary conditions?
c) Show that the $v = 0$ everywhere.
d) From the x-momentum equation, determine the u-velocity profile.
e) Noting that an enclosure is a closed system (*i.e.* no net flow rate), determine the resulting pressure gradient in terms of ρ, μ, U_p, h and g.
f) What is the magnitude and direction of the shear stress at the walls?

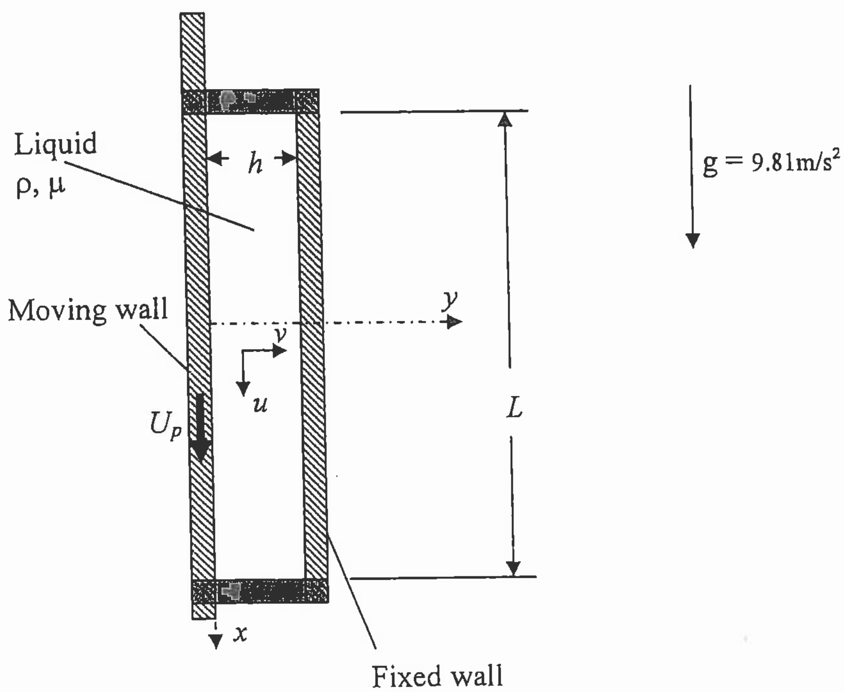
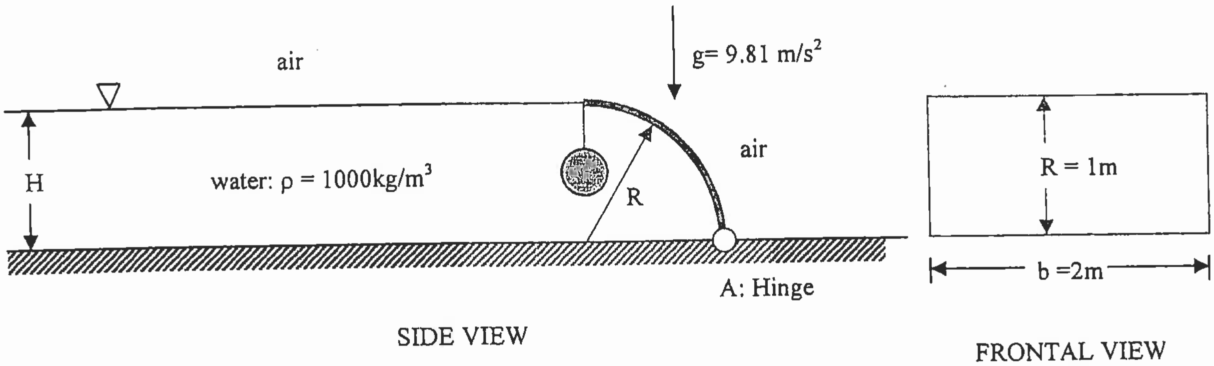
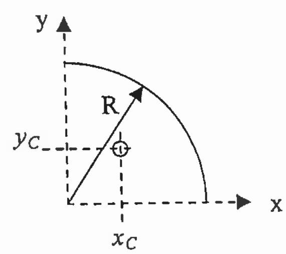


Figure 6: Schematic representation of thin enclosure filled with liquid and one wall moving at a constant rate.

Question 7: A gate, as shown in the figure below, is used to hold flood water into a reservoir. The gate is kept closed by a suspended spherical mass. The gate has a uniform width of b=2m (into the page) and a radius of R=1m. If the density of the sphere is 5500 kg/m^3, determine the minimum diameter of the sphere to keep the gate closed at the maximum level (i.e. when H=R). The gate is hinged at A. The mass of the gate may be assumed negligible.



Additional Information:



$$x_C = y_C = \frac{4R}{3\pi}$$

$$I_{xxC} = I_{yyC} = 0.05488R^4$$

$$I_{xyC} = -0.016471R^4$$

Volume of a sphere: $= \dfrac{\pi d^3}{6}$ where d is the diameter of the sphere.

Figure 7: Schematic representation of gate under hydrostatic loading. R=1m, b=2m.

MAJOR LOSSES

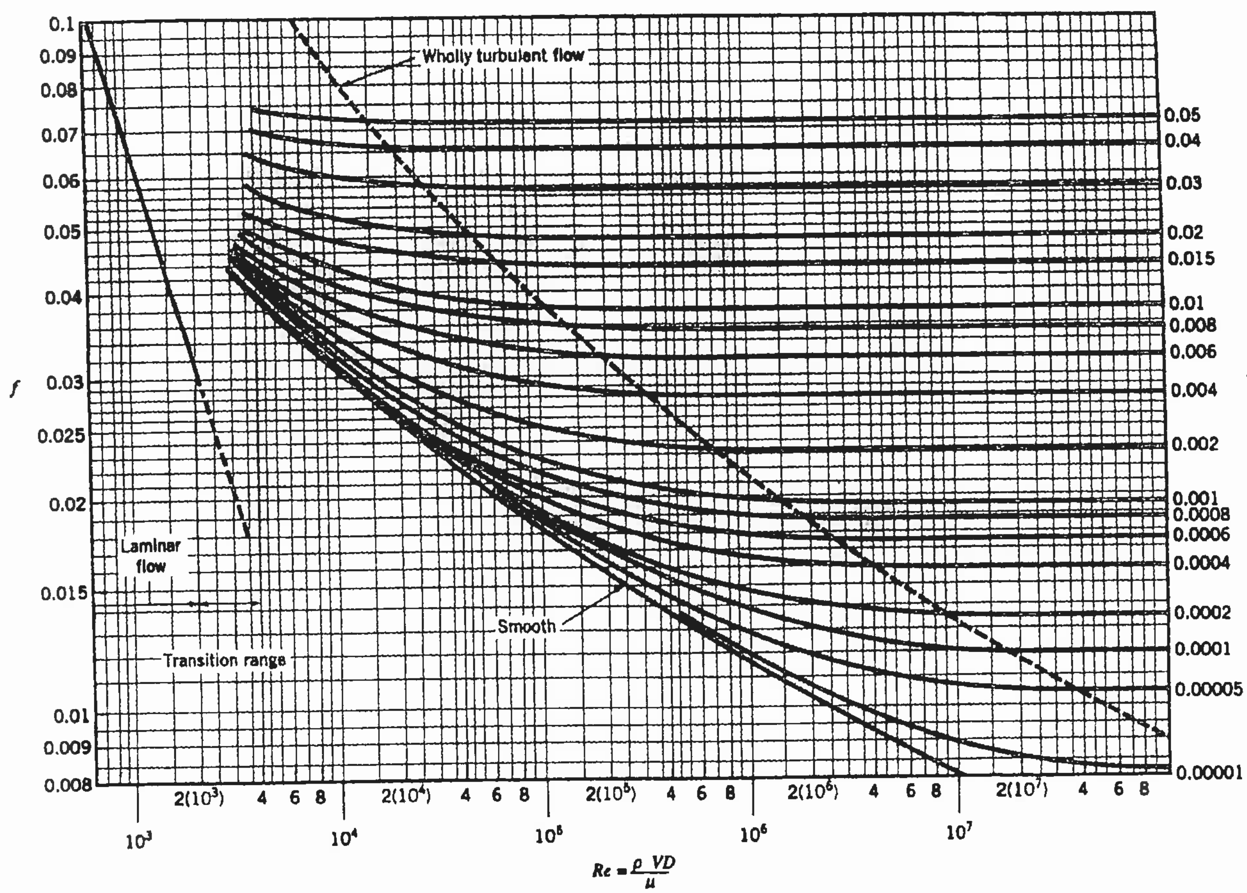
FRICTION COEFFICIENTS:

Colebrook Equation: $$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right]$$

Laminar Flow Equation: $$f = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} < 2000$$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

MOODY CHART for PIPE FRICTION



MINOR LOSSES

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210