

National Exams

**98-Comp-B6 Computer Control and Robotics**

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM.  
Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.  
The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.

1. (20 marks)

Let the Z-transform of  $f(k)$  be denoted  $F(z)$ .

a) Find the Z-transform of the following sampled signal.

$$f(k) = (0.8)^k - 4(0.4)^k, \quad k = 0, 1, 2, \dots$$

b) Compute the inverse Z-transform of

$$F(z) = \frac{2z^2 - 3z}{z^2 - 0.7z + 0.1}$$

2. (20 marks)

Let the Z-transform of  $f(k)$  be denoted  $F(z)$ . Use the Z-transform method to find the solution of the following difference equation.

$$y(k+2) - 0.7y(k+1) + 0.1y(k) = 0, \quad k = 0, 1, 2, \dots$$

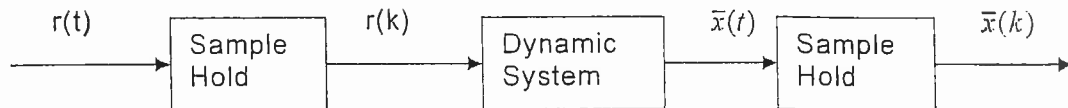
The initial conditions are  $y(1) = -1.6$ ,  $y(0) = 2$ .

Hint: Use the property  $Z\{f(k+n)\} = z^n F(z) - z^n f(0) - z^{n-1} f(1) - \dots - z f(n-1)$ .

## 3. (20 marks)

A dynamic system has the following continuous time model.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} r(t)$$



Write the discrete time equation that describes this system in the form,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = A \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + Br(k)$$

where  $A$  is a  $2 \times 2$  matrix and  $B$  is a  $2 \times 1$  vector. The sampling rate is 10 Hz. You may use either the approximate numerical method (do not use more than three terms) or the Laplace transform method. The candidate needs to find the values for the matrices  $A$  and  $B$ .

## 4. (20 marks)

The equation for a dynamic system given in discrete time is, (sampling period is  $T=0.1$  seconds).

$$\bar{x}(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.81 & 1.8 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

Design a full state feedback control law of the form,

$$u(k) = -K\bar{x}(k)$$

such that the controlled system has closed loop pole location in the  $z$ -plane at  $z = 0.7$  and  $z = 0.7$  (both poles at 0.7).

## 5. (20 marks)

Design a full order observer for the system,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & -0.81 \\ 1 & 1.8 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

where the output is,

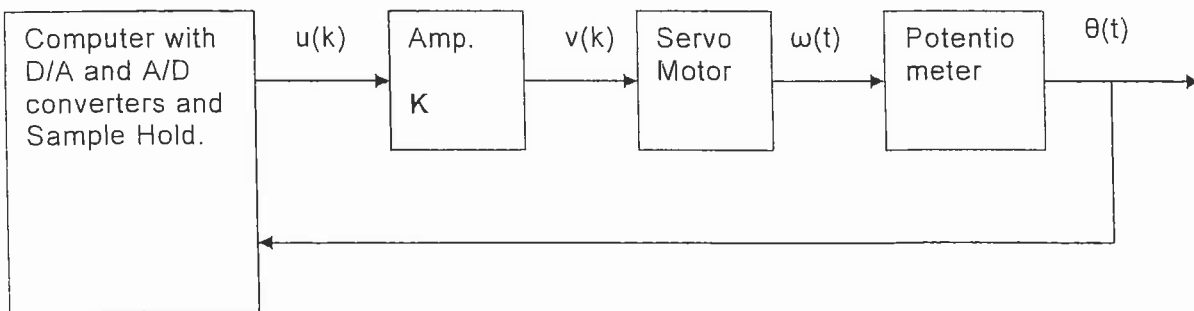
$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

The sampling time for the system is  $T = 0.1$  seconds. It is specified that the desired error dynamics be critically damped (i.e. the damping ratio is one), and have an undamped natural frequency of 4.0 rad/s. Convert the continuous time dynamic specifications into the discrete time z-domain. Where are the z-domain pole locations? What are the observer gains,  $K$ ? Write the observer in the form,

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(y - C\hat{x}(k))$$

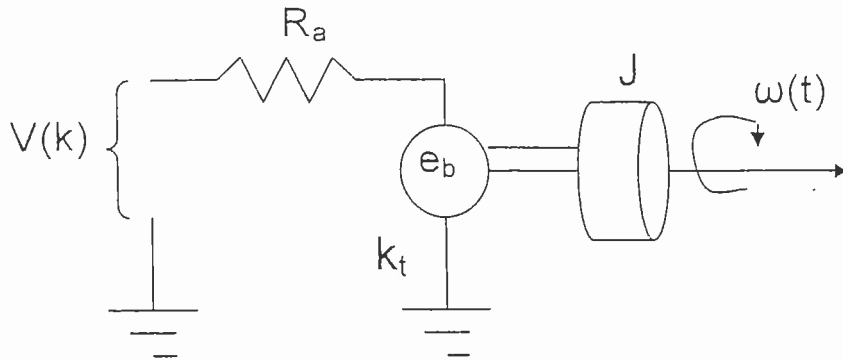
## 6. (20 marks)

You are given the following system to control with a digital computer. You are to design a position controller for the servo shown below.



Where,  $u(k)$  is the output of the computer's D/A converter and sample hold. The voltage  $v(k)$  is the output of the amplifier (i.e.  $v(k) = Ku(k)$ ). The angular velocity of the servo motor is  $\omega(t)$  and the output of the potentiometer is the

angular position given by  $\theta(t)$ . You have a model of the servo motor given by the diagram,



Where  $k_t$  is the motor torque constant in N-m/Amp,,  $J$  is the polar moment of inertia,  $R$  is the armature resistance. You are told that the motor must go to its commanded position within  $\frac{1}{2}$  sec and hold that position with a 0.5 degree accuracy. We know the following values,  $k_t = 1.0$  N-m/Amp,  $J = 0.1$  kg·m<sup>2</sup> and  $R = 10$   $\Omega$ . Discuss your design consideration with regard to the following issues,

- 1) Design Specifications.
- 2) Closed loop and open loop pole locations.
- 3) Sampling time.
- 4) Resolution of D/A and A/D converters.
- 5) Control strategy, will proportional position feedback control work? Why?

Justify your answers based on numerical calculations and assumptions.