National Exams December 2011
04-BS-1, Mathematics
3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. NO CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 12 marks, (b) 8 marks
2. (a) 10 marks, (b) 10 marks
3. 20 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. (a) 3 marks, (b) 3 marks, (c) 14 marks
8. 20 marks

1. Let P be the plane passing through the three points (0,1,2), (1,3,-1) and (2,0,1).
   (a) Find an equation representing the plane P.
   (b) Find the line of intersection between the plane P and the plane
   $$x - 2y + z = 3$$

2. (a) Find the eigenvalues and the eigenvectors of the matrix
   $$\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$$
   (b) Solve the system of differential equations
   $$\frac{dx}{dt} = 3x - 2y,$$
   $$\frac{dy}{dt} = x + y + e^{-2t}.$$
   subject to the initial conditions $x(0) = 2$, $y(0) = -1$.

3. Find the solution, $y(x)$, of the differential equation
   $$y'' + 9y = \sec 3x,$$
   $y'(0) = 0$, $y(0) = 1$. Note that $'$ denotes differentiation with respect to x.

4. Find an equation for the line tangent to the intersection of the surfaces
   $$x^2 + y^2 - 6z = 11$$
   and
   $$4x^2 + y^2 + z^2 - 4y - 4z + 3 = 0$$
   at the point $(1, 0, 2)$.

5. At what angle does the line represented parametrically by $x = 2 - t,\ y = t,\ z = 2 + 2t$ intersect the hyperboloid $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

6. Let S be the surface of the region defined by $x^2 + 4y^2 \leq 1,\ x \geq 0,\ y \geq 0,\ 0 \leq z \leq 4,$ and let F be the vector function $F(x, y, z) = (y^3, x^3, z^3)$. Evaluate the integral of F over the surface S.

7. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z = 1 + y - 2x$, travelled clockwise as viewed from the positive z-axis, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$.

   (a) Evaluate the divergence of $\mathbf{v}$

   (b) Evaluate the curl of $\mathbf{v}$

   (c) Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$.

8. Find the general solution of the differential equation

$$x^2y'' - 4xy' + 6y = 3x^4.$$

Note that $'$ denotes differentiation with respect to x.