National Exams – December 2011
07-Elec-B10, Electro-Optical Engineering

3 hours duration

NOTES:

1. If doubt exists as to the proper interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement about any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.
3. This is a **"Closed-Book"** examination. The candidate may have a single 8.5 inch by 11 inch sheet (both sides) of hand-written notes as an aid for the examination.
4. Any **five** questions constitute a complete paper. Only the first five questions as they appcar in your answer book will be marked.
5. All questions are of cqual value.
6. This examination paper has 4 pages.

Values of common constants:

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
$\mu_0 = 4\pi \times 10^{-7}$ H/m
$c = 2.998 \times 10^{8}$ m/s
$q = 1.602 \times 10^{-19}$ C
$h = 6.626 \times 10^{-34}$ J·s
$K = 1.381 \times 10^{-23}$ J/°K
0°K = -273°C
1 Å = 1.0×10^{-10} m

Si $\varepsilon_r = 11.8$
Si $n = 3.42$
Si $E_g = 1.11$ eV
Ge $\varepsilon_r = 16.0$
Ge $n = 4.01$
Ge $E_g = 0.67$ eV
GaAs $\varepsilon_r = 13.2$
GaAs $n = 3.63$
GaAs $E_g = 1.41$ eV
InGaAsP $n = 3.5$

$LiNbO_3$ $\varepsilon_r = 32$
$LiNbO_3$ $r = 30$ pm/V
$LiNbO_3$ $n_o = 2.30$

Useful formulas: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$ $\qquad P(n) = \frac{N^n \exp(-N)}{n!}$

$Al_xGa_{1-x}As \quad E_g(eV) = 1.424 + 1.266x + 0.266x^2$

$I_x = R_o\sqrt{P_o P_1}\cos\theta \qquad n(E) = n_0 - \frac{1}{2} r n_0^{\,3} E \qquad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Question 1

Consider a multi-mode step index fiber having core radius a, core index n_1 and cladding index n_2 where $n_2<n_1$ with very small index difference Δn.

(a) Derive the numerical aperture of the fiber for a ray entering the end face from air at angle θ from the fiber axis.

(b) Using geometrical optics, derive an expression for the pulse broadening $\Delta\tau$ due to the modal dispersion over a long length L of the fiber.

(c) If a= 50 μm, n_1=1.51 and n_2=1.50 what is the numerical aperture of the fiber, the critical angle of the fiber, and how much pulse broadening occurs over a fiber of length 5km?

(d) Estimate the maximum bit rate one could transmit over the fiber in (c) if the source is a laser diode at 1550 nm with narrow width and RZ signaling is used for the digital signal. Assume that the light pulse output is nearly rectangular.

(e) Why would a 5 km long GRIN fiber with radius a=50 μm show substantial improvement in the maximum bit rate one could transmit?

(f) For a fiber with core index n_1=1.51 and cladding index n_2=1.50, what would its radius need to be for single mode operation at 1550nm?

Question 2

A InGaAsP semiconductor diode laser designed for use at λ_0=1550 nm wavelength has the following parameters:

Cavity dimensions: length L=600 μm, width W=8 μm, depth D=2 μm.

Scattering and absorption losses: α_s=500 m^{-1}

Laser gain: $g(\lambda) = 4000\exp\left[\frac{-(\lambda-\lambda_0)^2}{2\sigma^2}\right]\ m^{-1}$ *where* $\sigma = 5.0\,nm$

Threshold current density: I_{th}=25 mA

(a) Calculate the two wavelengths closest to 1550 nm at which the laser will operate.

(b) What is the total loss and the photon lifetime in the laser cavity?

(c) How many modes will appear in the laser output?

(d) Calculate the photon density at a dc current level of 40 mA.

(e) What is the output power at 40 mA dc current.

Question 3

A fiber optic system is being designed to operate over a distance of 40 km. The following components are used.

Source: Laser diode at wavelength 900 nm with output power 10mW.

Fiber: Multimode step index silica fiber with radius 50 μm and attenuation of 3dB/km at 900 nm. The core and cladding indices are 1.500 and 1.495, respectively.

Detector: Avalanche diode circuit with bandwidth 1GHz.

Question 3 (Continued)

The analog baseband signal having bandwidth 100 kHz is sampled at the Nyquist rate and is converted to a digital NRZ signal with 9 bits per symbol. The maximum pulse broadening to be allowed is half the bit period. The coupling loss between the laser chip and the fiber is 3dB, and there is 2 dB coupling loss between the fiber and the detector. The detector needs an average minimum 400 photons/bit to obtain the specified BER of 1×10^{-12}. A security margin of 5dB is required for this BER.

(a) What semiconductor material should be used for the detector diode? What semiconductor material should be used for the laser diode? Explain your answers.
(b) Sketch the loss versus wavelength curve for a silica fiber and label all the important features.
(c) How many modes will propagate in the fiber?
(d) Determine the maximum link length due to the dispersion limit.
(e) Determine the maximum link length due to the attenuation limit.
 What do you conclude from (d) and (e)?
(f) Explain what a "3R" repeater is. If a "3R" repeater uses the same laser diode and detector as specified above, how many repeaters are required for the 40 km link?

Question 4

(a) Describe the advantages and disadvantages of using
 (i) a PIN diode photodetector
 (ii) an avalanche diode photodetector.
(b) A PIN photodetector is operated in the photoconductive mode with a load resistor R. Sketch the I-V response of the detector and the operating load line.
(c) A PIN diode photodetector has depletion layer capacitance C and operates with a load resistor R. The depletion layer has width W. The diode is operated in the photoconductive mode at reverse bias high enough for the carriers to reach drift velocity saturation v_s. Write the expression for the total response time of the detector circuit.
(d) From your expression in (c), find the optimum width of the depletion region to minimize response time. (Neglect limitations due to diffusion of charge carriers).
(e) A GaAs PIN diode circuit has the following parameters:
 Detector Area 10μm x 10μm.
 Carrier saturated drift velocity 1.8×10^7 cm/s
 Load resistor 50Ω
 Calculate the optimum response time of this circuit.
(f) Explain the physical reason for an optimum depletion layer width.

Question 5

An optical fiber link is being designed for 2.5Gb/s NRZ data at 1550nm. An APD photodetector is used. The APD has dark current 1nA, quantum efficiency 0.80, multiplication factor M, and excess noise factor $M^{0.5}$. A load resistor 500Ω is used and the circuit operates at 27^0C. The bandwidth of the detector circuit must be 0.7B where B is the bit rate. The optical signal power at the diode is -35dBm.

(a) Write the expression for the SNR of the detector circuit without substituting in the detector parameter values. (Neglect any noise due to an amplifier).

(b) Using your expression in (a), determine an expression for M that optimizes the SNR. For the given diode circuit what is the numerical value of M?

(c) If M is 18, what is the SNR of the detector circuit?

(d) What noise contribution dominates in (c)?

Question 6

(a) Describe the operation of a planar integrated Mach Zehnder light intensity modulator . Use a sketch of the device to aid your answer.

(b) Describe the operation of a longitudinal Pockel cell modulator. Use a sketch to show all the components of the device as an aid to your answer.

(c) Draw the modulation characteristic (output light intensity versus applied voltage) of the Pockel cell modulator.

(d) A longitudinal Pockel cell modulator is made using a cylindrical $LiNbO_3$ crystal of length 40mm and diameter 20mm. It is used to modulate light at 635nm wavelength. The bandwidth of the modulator is limited by the RC response of the circuit where R is the load resistance and C is the capacitance of the Pockel cell. Calculate the half-wave voltage of the modulator.

(e) How much power is required to drive the modulator in (d) with a sinusoidal voltage source of maximum allowed amplitude and maximum frequency?