NATIONAL EXAMS December 2011
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. a Casio or Sharp approved calculator.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

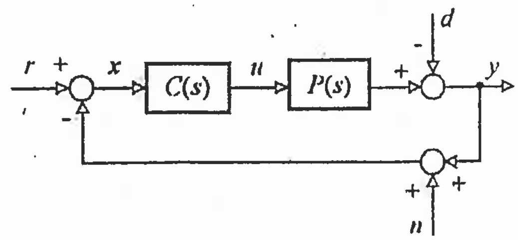
07-Elec-B2 Advanced Control Systems – December 2011

1. Consider the control system below with, $P(s)=\dfrac{30}{(s+3)(s+10)}$ and $C(s)=\dfrac{K_1+sK_2}{s}$.

(a) Let $K_2=0$. Determine a value of K_1 say $K_1=K_{1M}$ such that the closed loop system exhibits sustained oscillation. Determine the oscillation frequency.

(b) For $K_1=K_{1M}$ determine a value for K_2 such that the phase margin exceeds 45 degrees.

(c) Determine the steady state value of $y(t)$ when $d(t)$ = a ramp of slope 3, and $r(t)$ = a unit step, and $n(t)$ = 0.



2. Consider the system,

$$\dot{x}(t)=Ax(t)+Bu(t),\quad A=\begin{pmatrix}0&0&-3\\0&0&1-3\alpha\\0&1&0\end{pmatrix},\ B=\begin{pmatrix}1\\ \alpha\\0\end{pmatrix}$$

$$y(t)=Cx(t)+Du(t),\quad C=\begin{pmatrix}1&1&0\end{pmatrix},\ D=0$$

(a) Determine the conditions for controllability and observability.

(b) For $\alpha=2$, design a controller of the form $u(t)=Lr(t)-Kx(t)$, i.e., determine L and K such that the closed loop poles are $s=-2+j$, $s=-2-j$, $s=-4$, and the steady state tracking error, $e=r-y$, is zero when $r(t)$ is a unit step.

3. The discrete time model for a system has the form, $y(z)=P(z)u(z)$ where $P(z)=\dfrac{4z^{-2}}{1-z^{-1}}$.

(a) Consider a discrete proportional feedback controller, K. Determine the range of K that ensures closed loop stability.

(b) Draw the associated root locus.

(c) Assuming $y(k)$ is sampled uniformly from the output of a continuous linear time invariant system and the sample $u(k)$ is applied uniformly to a ZOH that drives the input of the continuous time system, determine a transfer function for the continuous time system assuming the sample period is $h=1$s.

4. Several experiments are conducted on an unknown system, $P(s)$:

- When a step of magnitude 5 is applied to the input, the steady state output is 20 and the maximum overshoot is 30.
- When a sinusoid of frequency 8 rad/sec is applied, the phase lag at the output is 90°.

(a) Assume the system, $P(s)$, is second order system and has no finite zeros. Find the parameters of the second order model.

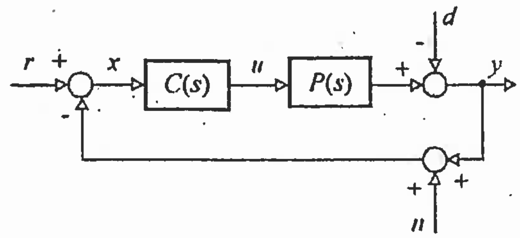
(b) Justify whether or not it is feasible to reduce the percentage overshoot by using a proportional feedback controller.

5. Consider the feedback system below with $P(s)=\dfrac{4(s+3)}{s^2+0.2s+2}$.

(a) Determine a feedback controller, $C(s)$, such that the closed loop transfer function relating r to y is given by $\dfrac{32}{8s^2+3s+32}$. Note: $C(s)$ must be proper, i.e., the degree of the numerator must be greater than or equal to the degree of the numerator.

(b) Determine the gain and phase margin of the feedback design.

(c) Determine the steady state value of u when d is a unit step, $r = 0$, and $n = 0$.

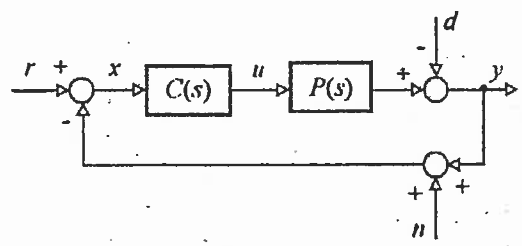


6. Consider the feedback system below with, $C(s)=\dfrac{K}{s}$, $P(s)=\dfrac{20e^{-s}}{s+10}$.

(a) Design $C(s)$ such that the gain margin is 6dB.

(b) Determine the phase margin.

(c) The system is operating with constant inputs, $n(t) = 0$, $d(t) = d_0$, and $r(t) = r_0$. Determine the steady state tracking error, $e(t) = r(t) - y(t)$, as a function of K.



Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\frac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\frac{Kz}{z-a}$	Ka^n
$\frac{(C+jD)z}{z-re^{j\varphi}}+\frac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^n\left(C\cos n\varphi-D\sin n\varphi\right)$
$\frac{Kz}{(z-a)^r}$, $r=2,3...$	$\frac{Kn(n-1)...(n-r+2)}{(r-1)!a^{r-1}}a^n$

Table of Laplace and z-Transforms
(*h* denotes the sample period)

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos\beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin\beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos\beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin\beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$