National Exams December 2011

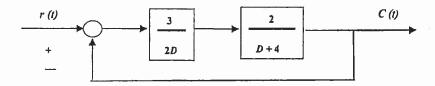
07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

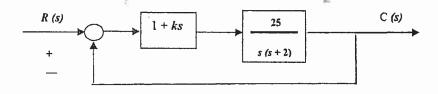
NOTES:

- 1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
- 2. Candidates may use a Casio <u>or</u> Sharp approved calculator. This is a <u>closed book</u> exam. No aids other than semi-log graph papers are permitted.
- 3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
- 4. All questions are of equal value.

1. Determine the position, velocity, and acceleration error constants, and the steady-state error to a unit step, a unit ramp, and a unit parabolic input for the system shown in the figure below.



2. To improve the transient behaviour of a system, a controller with proportional and derivative action is added as shown in the figure below. Determine the value of k such that the resulting system will have a damping ratio of 0.5. What is the response c(t) of this resulting system to a unit step function r(t) when all initial conditions are zero?



3. Determine the maximum value for the Bode gain K_B which will result in a gain margin of 6 db or more and a phase margin of 45° or more for the system with the open-loop frequency response function

$$GH(j\omega) = \frac{K_B}{j\omega (1 + j\omega/5)^2}$$

4. a) Using the Laplace transform technique, find the transient and steady-state responses of the system described by the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 1$ with initial conditions

$$y(0^+)$$
 and \underline{dy} = 1.

b) Using the Laplace transform technique, find the unit impulse response of the system described by the differential equation $\underline{d}^3y + \underline{d}y = x$.

5. Draw the Bode diagram representation of the frequency response for the transfer functions given by:

a) GH (s) =
$$\frac{(s+3)}{(s^2+4s+16)}$$

b) GH (s) =
$$(1 + 0.5s)$$

6. Draw the root locus for the following open-loop transfer function.

$$GH(s) = \frac{K}{(s+1)(s^2+s+1)}$$

Determine the range of the gain for which the system is stable.

Laplace Transform Table

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1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$. Unit-step function u,(ε)
$\frac{1}{s^2}$	Unit-ramp function t
n 3 ^{x+1}	t* (n = positive integer)
$\frac{1}{s+\alpha}$	e ^{-w}
$\frac{1}{(s+\alpha)^i}$	te ^{-ar}
$\frac{n!}{(s+\alpha)^{n+1}}$	f^*e^{-at} (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta - \alpha} (e^{-\alpha} - e^{-\beta}) \ (\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta - \alpha} (\beta e^{-\beta r} - \alpha e^{-\alpha r}) (\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\mu})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^{2}}(1-e^{-\alpha}-\alpha te^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^{1}}(\alpha t - 1 + e^{-\alpha t})$
$\frac{x_{3}(x+\alpha)_{1}}{f}$	$\frac{1}{\alpha^{2}}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Leplace of an items (1)	Assistance Europellonia (1977) (1977)
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-at}$
$\frac{\omega_1^2}{s^2 + \omega_1^2}$	sin ω _τ (
$\frac{s}{s^2 + \omega_*^2}$	COS ω _α !
$\frac{\omega_s^1}{s(s^1+\omega_s^2)}$	1 – cos ω ₄ ι
$\frac{\omega_4^2(s+\alpha)}{s^2+\omega_4^2}$	$\omega_4 \sqrt{\alpha^7 + \omega_1^2} \sin(\omega_4 t + \theta)$ where $\theta = \tan^{-1}(\omega_4 / \alpha)$
$\frac{\omega_n}{(s+\alpha)(s^1+\omega_n^2)}$	$\frac{\omega_{a}}{\alpha^{1} + \omega_{a}^{1}} e^{-\alpha t} + \frac{1}{\sqrt{\alpha^{2} + \omega_{a}^{2}}} \sin(\omega_{a}t - \theta)$ where $\theta = \tan^{-1}(\omega_{a}/\alpha)$
$\frac{\omega_4^1}{s^1 + 2\xi\omega_4 s + \omega_4^1}$	$\frac{\omega_4}{\sqrt{1-\zeta^2}}e^{-i\omega_{\zeta}}\sin\omega_{\zeta}\sqrt{1-\zeta^2} (\zeta<1)$
$\frac{\omega_{A}^{2}}{s(s^{2}+2\zeta\omega_{A}s+\omega_{A}^{2})}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-t - \zeta} \sin(\omega_a \sqrt{1 - \zeta^2} 1 + \theta)$ where $\theta = \cos^{-1} \zeta$ ($\zeta < 1$)
$\frac{s\omega_a^1}{s^2+2\zeta\omega_a s+\omega_a^2}$	$\frac{-\omega_*^1}{\sqrt{1-\zeta^1}}e^{-\zeta\omega_*^1}\sin(\omega_*\sqrt{1-\zeta^2}i-\theta)$ where $\theta=\cos^{-1}\zeta$ ($\zeta<1$)
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_{a} \sqrt{\frac{\alpha^{2} - 2\alpha\zeta\omega_{4} + \omega_{4}^{2}}{1 - \zeta^{2}}} e^{-i\omega_{1}} \sin(\omega_{a} \sqrt{1 - \zeta^{2}} t + \theta)$ where $\theta = (an^{-1} \frac{\omega_{c} \sqrt{1 - \zeta^{2}}}{\alpha - \zeta\omega_{a}})$ ($\zeta < 1$)
$\frac{\omega_a^2}{s^2(s^2+2\zeta\omega_a s+\omega_a^2)}$	$i - \frac{2\zeta}{\omega_a} + \frac{1}{\omega_a^2 \sqrt{1 - \zeta^2}} e^{-i\omega_t} \sin(\omega_a \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1)$ ($\zeta < 1$)



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