National Exams December 2011

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. Answer any **3** of the 4 questions in Part **A** and any **2** of the 3 questions in Part **B**
4. Weighting: Each question is equally weighted within a section.
   Part **A**: 40%; Part **B** 60%

Part A: Answer any **3** of the following 4 questions.

Question A1: A water turbine is designed to produce 27 MW when running at 93.7 rpm under a head of 16.5 m. A model turbine with an output of 37.5 kW is to be tested under dynamically (Froude) similar conditions with a head of 4.9 m. Calculate the model speed and scale ratio. Assuming a model efficiency of 88%, estimate the volume flow rate through the model. It is estimated that the force on the thrust bearing of the full-size machine will be 7.0 GN. For what thrust must the model bearing be designed? For water, $\rho = 1000$ kg/m^3 and $g = 9.81$ m/s^2.

Question A2: Air flows from a reservoir where the pressure is 300 kPa and the temperature is 500 K through a throat of a convergent-divergent nozzle. Downstream of the throat, there is a normal shock. The area of the throat is 1 m^2. At the location of the shock, the area is 2 m^2.

a) Determine the Mach number, static pressure and flow speed directly upstream of the shock. What is the mass flow rate?
b) Determine the Mach number, the mass flow rate, the stagnation temperature and pressure, the static pressure and the flow speed directly downstream of the shock.

For air: R = 287 J/kg-K; $\gamma = 1.4$.

Question A3: Air at 20°C and 1 atm flows at 20 m/s past a flat plate as shown in the figure below. A pitot tube is used to read the stagnation pressure. The tube reads the pressure at a location $x = 0.90$ m downstream of the plate leading edge and 2 mm above the plate. If the manometer is filled with Meriam oil (SG = 0.827), estimate the manometer head, h. Assume laminar flow. For air: $\nu = 1.5 \times 10^{-5}$ m^2/s; $\rho = 1.23$ kg/m^3.

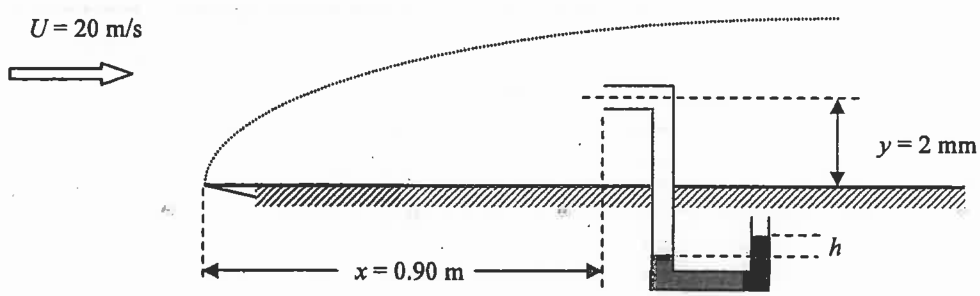


Figure A3: Schematic of boundary layer past plate with pitot tube and manometer.

Question A4: A pitot-static tube is introduced in a supersonic flow. Assuming that a normal shock is generated upstream of the tube, determine the speed at which the gas is flowing (i.e. upstream of the shock) if the tube reads a stagnation pressure of 130 kPa, a static pressure of 110 kPa and a static temperature of 519K, if: (a) the gas is air (R = 287 J/kg-K, $\gamma = 1.4$); (b) the gas is Helium (R = 2077 J/kg-K, $\gamma = 5/3$).

Part B: Answer any **2** of the following 3 questions.

Question B1: The influence of a helicopter, hovering over the ground at an elevation h in still air of density ρ, can be modelled using potential theory. The helicopter is replaced by a dipole, of strength λ, located h above the ground.

a) Determine the stream function used to model this flow.
b) Verify that the ground is effectively modelled (i.e. no flow crosses the ground).
c) Determine the mathematical expression for the pressure distribution along the ground in terms of λ, ρ, P_∞, h and x.

Neglect any gravitational effects. P_∞ is the pressure of the still air far away from the ground.

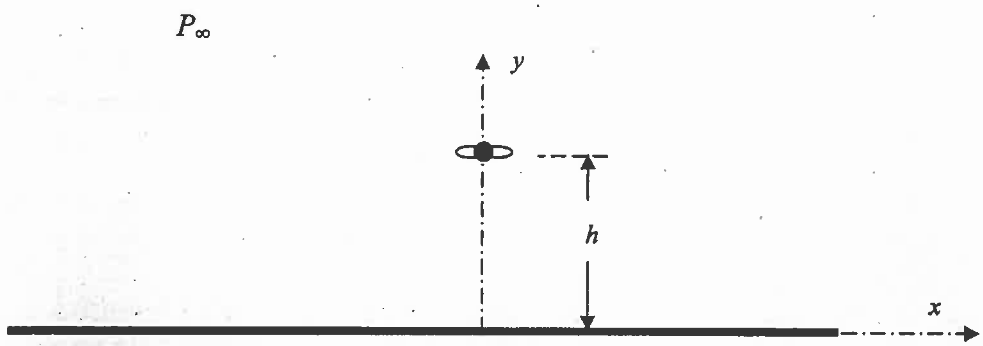


Figure B1: Schematic of helicopter hovering over ground at height h.

Question B2: Consider an infinite, solid cylinder of radius R placed in a still, infinite, constant property (ρ, μ) Newtonian fluid. If the cylinder rotates about its axis at a constant angular speed Ω, determine:

a) The radial, v_r, and the circumferential, v_θ velocity distribution in the fluid. State your assumptions clearly.
b) The torque needed to maintain the cylinder at constant Ω.
c) The circulation about the cylinder.
d) Whether the fluid flow can be modelled using potential flow theory and provide a justification.

Hint:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)-\frac{v}{r^2}=\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v)\right)$$

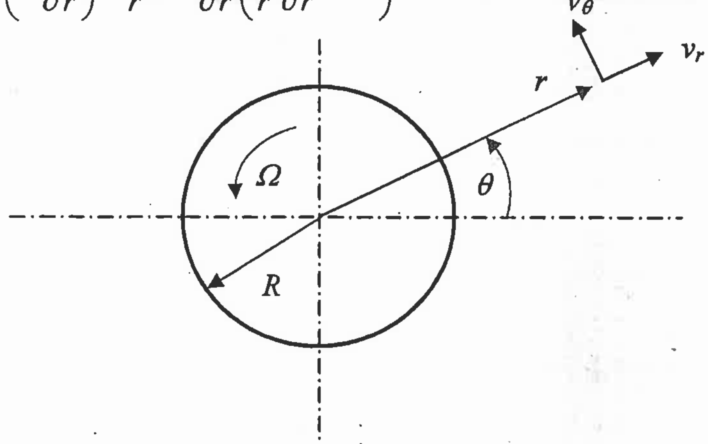
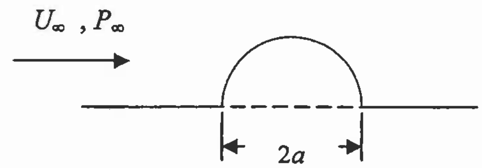


Figure B2: Cylinder rotating in a Newtonian fluid.

Question B3: An arctic hut is shaped like a long half cylinder. The flow field arising around it is described by the stream-function:

$$\Psi = U_\infty \left(1 - \frac{a^2}{r^2}\right) r \sin\theta$$



where U_∞ is the wind speed and P_∞ is the pressure far upstream, a is the radius of the hut.

a) Sketch the flow, clearly indicating the physical boundaries (i.e. the streamlines and corresponding stream-function values used to model the walls and ground).

b) Determine the pressure coefficient $C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho U_\infty^2}$ on the surface of the hut and along the ground upstream and downstream of the hut. Where are the stagnation points located? Where is the lowest pressure? What is the value of the pressure coefficient at that point?

c) If there is a hole in the roof at the location $\theta = 30°$, what is the pressure inside the hut? What must be the mass of the roof (per unit length) to keep the hut anchored? You are given that the wind speed far upstream is 30 km/hr, the air density is 1.23 kg/m^3 and the hut diameter is 5 m.

Aid Sheets

Blasius Profile Approximations:

$$\frac{u}{U_\infty} \cong 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad\qquad \frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

Compressible Flow:

Adiabatic flow:

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow:

$$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \quad ; \quad \frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

$$\dot{m} = \rho U A = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_o}{\sqrt{T_o}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma + 1}{2(\gamma - 1)}} A$$

Shock Relations:

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1{}^2}{1 + \gamma M_2{}^2} = \frac{2\gamma M_1{}^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \qquad M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

Conservation Equations for Cylindrical-polar Coordinate system

$$\overrightarrow{U} = v_r \overrightarrow{e_r} + v_\theta \overrightarrow{e_\theta} + v_z \overrightarrow{e_z}$$

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Linear Momentum Equations:

***r*-momentum:**

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right]$$
$$= -\frac{\partial P}{\partial r} + \rho g_r + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{r\theta}) + \frac{\partial}{\partial z}(\tau_{rz}) - \frac{\tau_{\theta\theta}}{r}$$

θ-momentum:

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right]$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \rho g_\theta + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \tau_{\theta r}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}) + \frac{\partial}{\partial z}(\tau_{\theta z})$$

***z*-momentum:**

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right]$$
$$= -\frac{\partial P}{\partial z} + \rho g_z + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{zr}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{z\theta}) + \frac{\partial}{\partial z}(\tau_{zz})$$

$$\tau_{rr} = \mu\left(2\frac{\partial v_r}{\partial r} - \frac{2}{3}\nabla\cdot\vec{U}\right)$$

$$\tau_{\theta\theta} = \mu\left(2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}\nabla\cdot\vec{U}\right)$$

$$\tau_{zz} = \mu\left(2\frac{\partial v_z}{\partial z} - \frac{2}{3}\nabla\cdot\vec{U}\right)$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$$

$$\tau_{rz} = \tau_{zr} = \mu\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$$

$$\nabla\cdot\vec{U} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z)$$

$$\nabla\times\vec{U} = \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}\right)\vec{e_r} + \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right)\vec{e_\theta} + \left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)\vec{e_z}$$