National Exams December 2011

98-Phys-B4, Signals and Communications

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a Closed Book Exam but one aid sheet is allowed written on both sides. A Casio or Sharp approved calculator is permitted.

3. There are a total of six questions. Choose five questions based on your preference. Five (5) questions constitute a complete paper. The first five (5) questions as they appear in the answer book will be marked.

4. All questions are of equal value.

5. Clarity and organization of the answer are important.

1. **(20 marks total)**

   (a) **(10 marks)** Find the exponential Fourier series for the signal in Fig. 1(a).

   (b) **(5 marks)** Using the results in part (a), find the Fourier series for signal $x_1(t)$ in Fig. 1(b), which is a time-shifted version of the signal $x(t)$.

   (c) **(5 marks)** Using the results in part (a), find the Fourier series for the signal $x_2(t)$ in Fig. 1(c), which is time-scaled version of the signal $x(t)$.

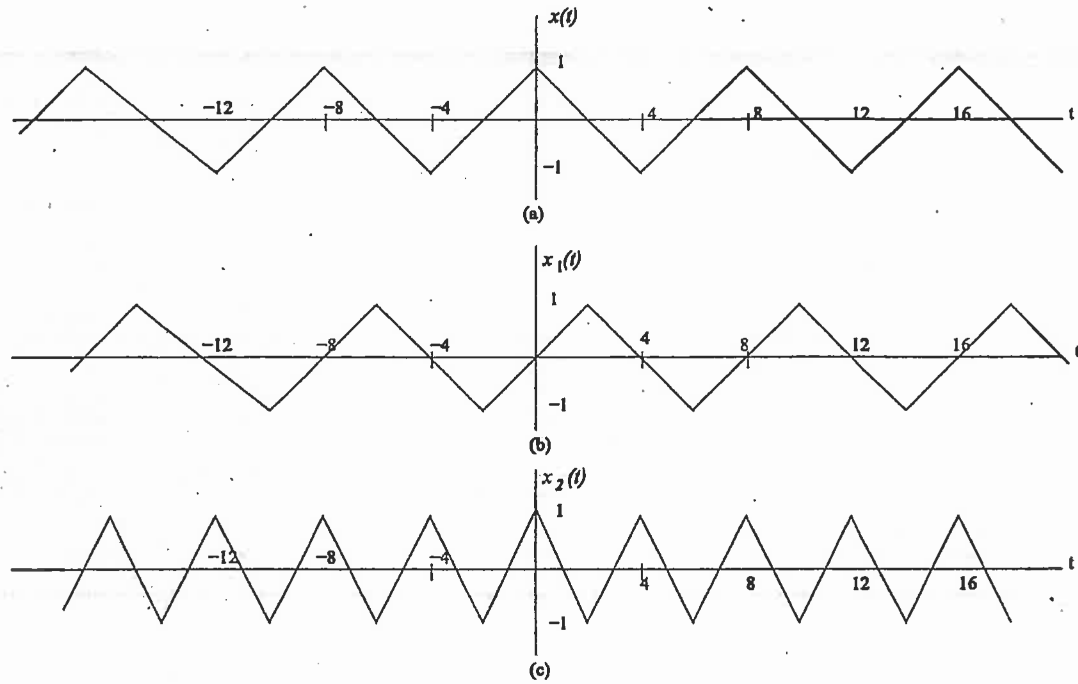

Figure 1:

2. **(20 marks)** A continuous time signal $x(t)$ is given as:

$$x(t) = 1 + \cos(\omega_0 t) + 2\sin(2\omega_0 t).$$

A linear time invariant continuous time system is shown in Fig. 2(a) where $x(t)$ and $y(t)$ are input and output respectively, $\delta_{T_s}(t) = \sum_{n=-\infty}^{n=\infty} \delta(t - nT_s)$ where $\delta(t)$ is unit impulse function and T_s is the sampling period. Signal $x(t)$ is sampled at frequency f_s Hz to produce $x_s(t)$. System response of $h(t)$ is shown in Fig. 2(b). Given that $f_s = \frac{5\omega_0}{2\pi}$,

(a) **(6 marks)** Find $X(\omega)$.

(b) **(8 marks)** Sketch $X_s(\omega)$ for $-8\omega_0 \le \omega \le 8\omega_0$.

(c) **(6 marks)** Find $y(t)$.

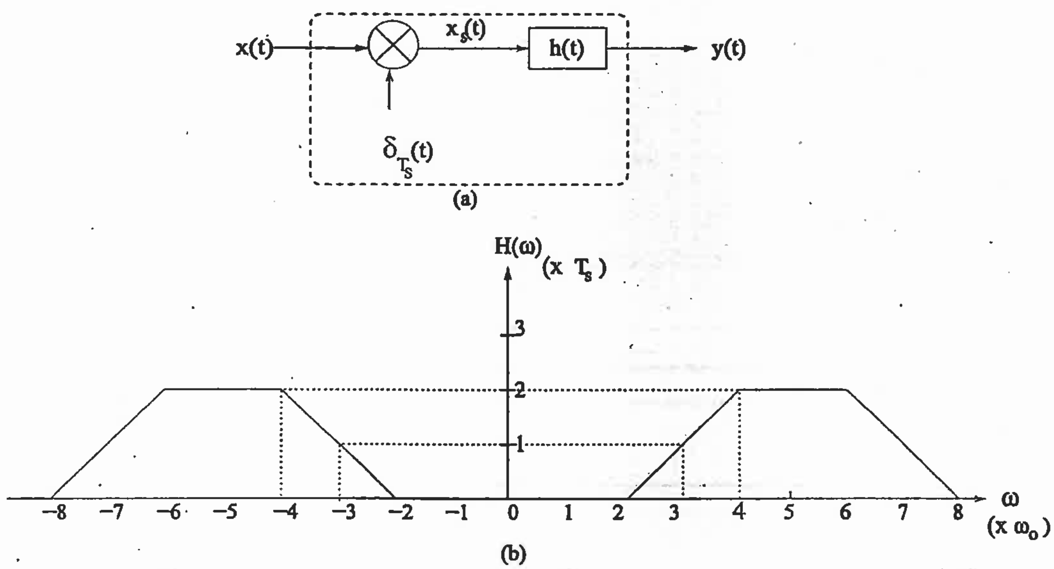

Figure 2:

3. **(20 marks total)** A LTI system is described by its impulse response:

$$h(t) = [e^{-t} - e^{-2t}]u(t).$$

(a) **(5 marks)** Find the frequency response of the system $H(j\omega)$.

(b) **(5 marks)** Using the result from (a) and the Fourier transform properties, find the differential equation that relates input $x(t)$ to output $y(t)$ in the above system.

(c) **(6 marks)** Plot the magnitude and phase response of $H(j\omega)$.

(d) **(4 marks)** Determine the output $y(t)$ when the input to the system is:

$$x(t) = 2 + \sqrt{18}\cos(\sqrt{2}t).$$

4. **(Total 20 marks)** Let $x(t) = cos(200\pi t)$ be used to modulate the carrier $\cos(4000\pi t)$ generate a DSBSC AM signal $x_M(t)$. The modulated signal $x_M(t)$ is demodulated using signal $cos[2\pi(2000 + \Delta f)t + \theta]$. For each case, explain whether (and how) the original signal can be reconstructed from the demodulated signal.

   (a) **(4 marks)** $\Delta f = 0, \quad \theta = 0$

   (b) **(4 marks)** $\Delta f = 0, \quad \theta = 0.25\pi$

   (c) **(4 marks)** $\Delta f = 0, \quad \theta = 0.5\pi$

   (d) **(4 marks)** $\Delta f = 10Hz, \quad \theta = 0$

   (e) **(4 marks)** $\Delta f = 10Hz, \quad \theta = 0.5\pi$

5. **(Total 20 marks)** A message signal $x(t)$ containing three unit-amplitude cosines at 200, 300, and 400 Hz is used to modulate a carrier at $f_c = 20$ kHz to generate the AM signals $x_1(t)$ and $x_2(t)$. Sketch the spectrum of each AM signal and compute the power in the sidebands as a fraction of the total power.

   (a) **(10 marks)** $x(t) = [1 + 0.2x(t)]\cos(2\pi f_c t)$

   (b) **(10 marks)** $x_2(t) = x(t)\cos(2\pi f_c t)$

6. **(20 marks total)** When the input to an LTI system is

$$x[n] = (\frac{1}{2})^n u[n] + (3)^n u[-n-1],$$

the corresponding out put is

$$y[n] = 5(\frac{1}{2})^n u[n] - 5(\frac{2}{3})^n u[n].$$

(a) **(8 marks)** Find the system function $H(z)$ of the system. Plot the pole(s) and zero(s) of $H(z)$ and indicate the region of convergence.

(b) **(4 marks)** Find the impulse response $h[n]$ of the system.

(c) **(4 marks)** Write a difference equation that is satisfied by the given input and output.

(d) **(4 marks)** Is the system stable? Is it causal?