National Exams May 2011

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

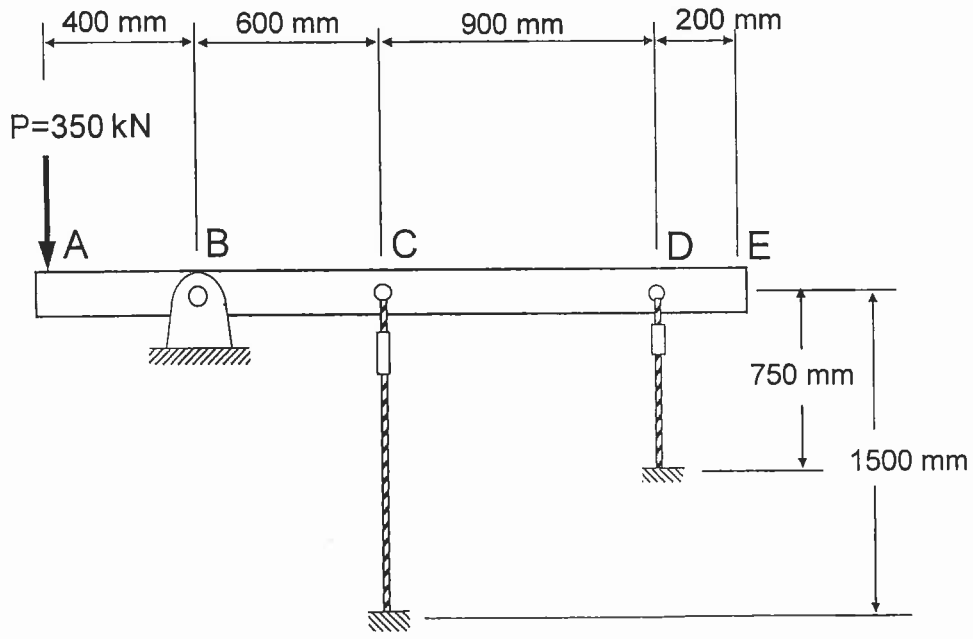
Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: A rigid bar ABCDE is loaded with a concentrated load P = 350 kN at A. The bar is supported by a 50 mm diameter pin at B and two 25 mm diameter cables at C and D. The cable at D has a length of 750 mm while the cable at C is twice as long with a length of 1500 mm. Both cables are made of steel with an elastic modulus of 200 GPa and yield strength of 350 MPa.

[12 marks] (a) determine the forces carried by each cable

[4 marks] (b) compute the corresponding displacement at the location where the load is applied (point A)

[4 marks] (c) compute the corresponding shear stress in the 50 mm diameter pin at B given that the pin is loaded in double shear

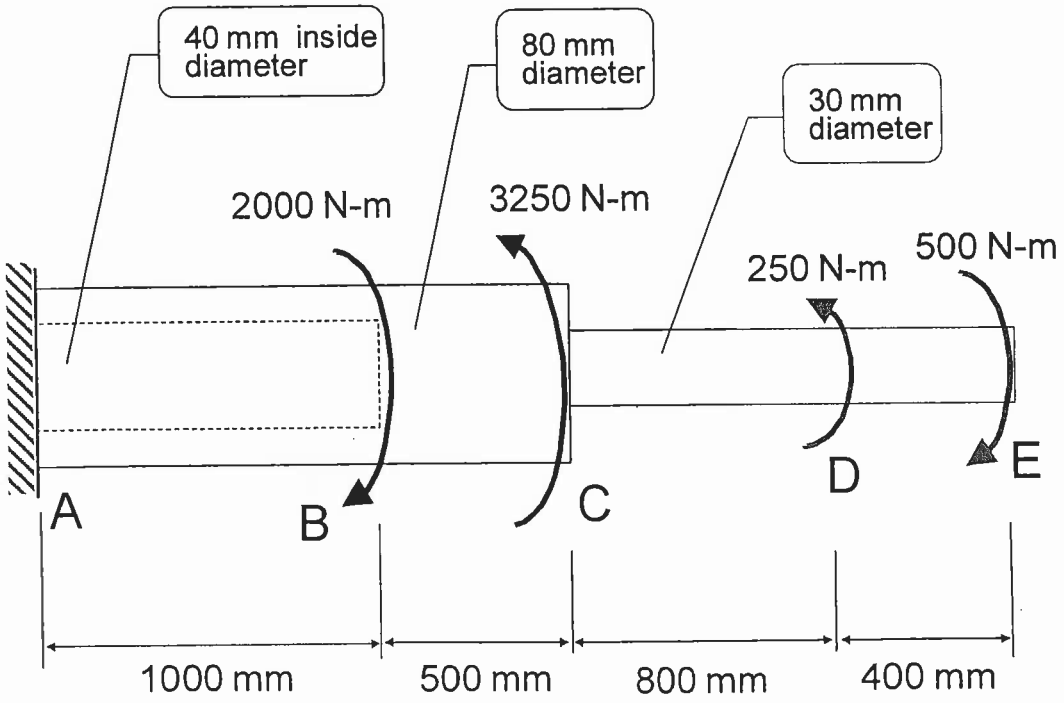


Question 2: A circular stepped shaft ABCDE is subjected to the torques shown. Part of the shaft AB is hollow. The shaft is made of aluminum with an elastic modulus E = 75 GPa, shear modulus G = 30 GPA, and yield stress (in shear) τ_y = 225 MPa.

[12 marks] (a) determine the maximum shear stress in the stepped shaft, and sketch the corresponding variation of shear stress (along the radius) on the cross-section of the shaft at this point.

[6 marks] (b) find the angle of twist at the end of the shaft (point E) and give your answer in degrees.

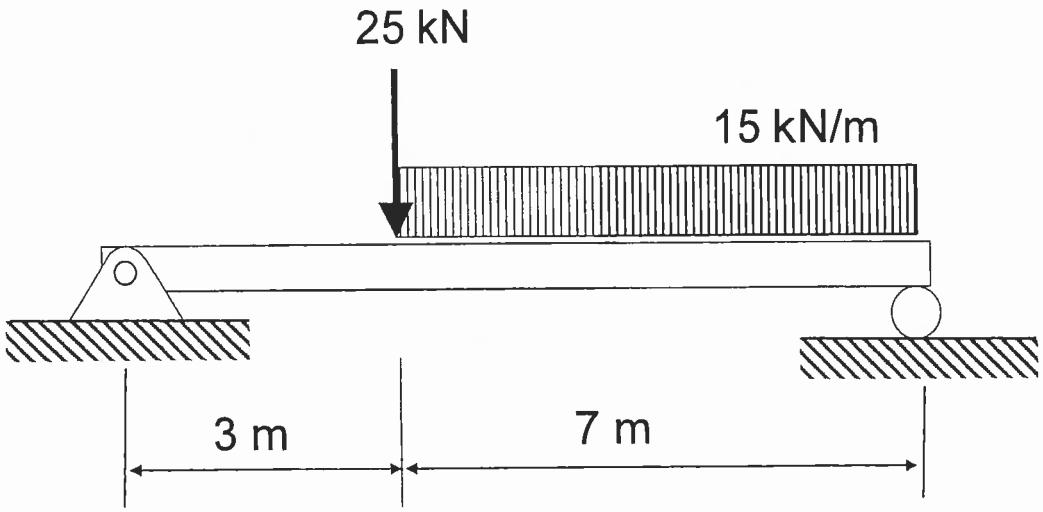
[2 marks] (c) what would happen if the loads on the shaft were tripled? Support your answer with calculations.



Question 3: A simply supported beam is subjected to a uniformly distributed load over part of the member span and a concentrated load as shown.

[20 marks] Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam. Next construct the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).



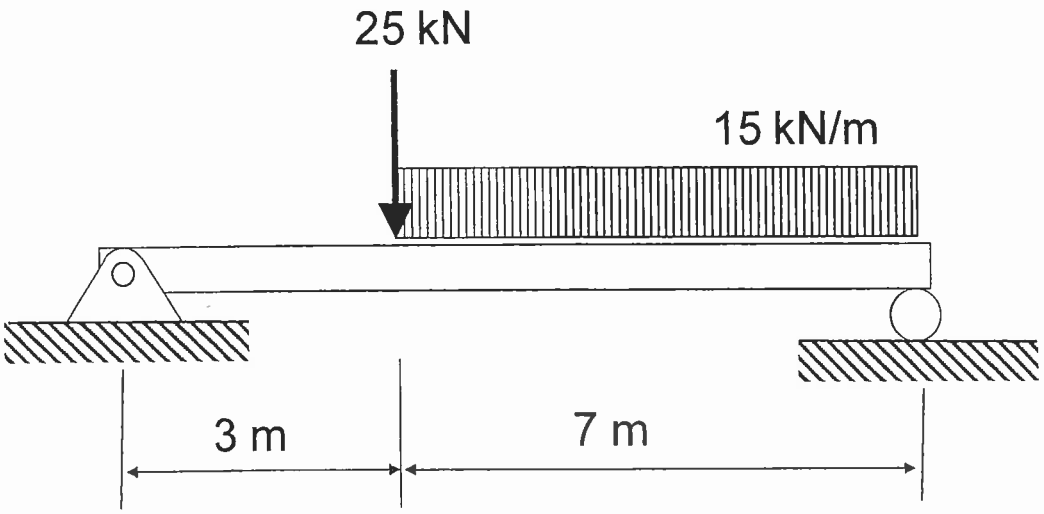
Note that **no credit** will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

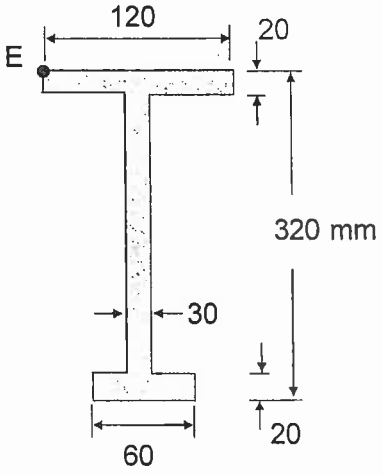
Question 4: A simply supported beam is subjected to a uniformly distributed load over part of the member span and a concentrated load as shown. The beam has the cross-section shown below and is made from steel having an allowable normal stress of 260 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel equals 200 GPa

[9 marks] (a) determine the maximum normal stress in the member

[9 marks] (b) determine the maximum shear stress in the member

[2 marks] (c) determine the shear stress at point E on the cross-section at the left support.



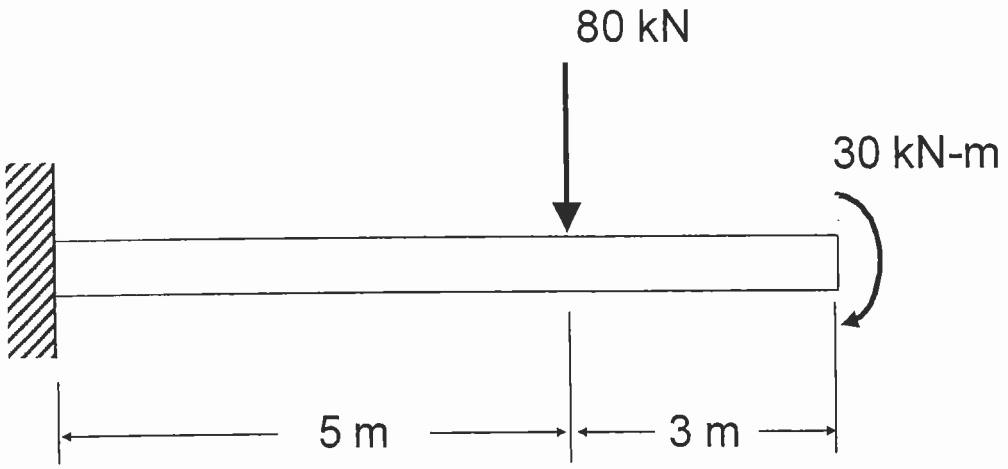


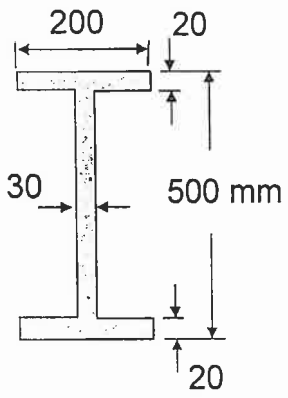
beam cross-section
(all dimensions in mm)

Question 5: A cantilevered beam (fixed at the left) supports a concentrated load of 80 kN acting 5 m from the fixed support and a 30 kN-m couple acting at the free end of the beam as shown. The beam has the cross-section shown below and is made from steel having an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel used equals 200 GPa.

[18 marks] (a) Determine the deflection and slope at the free end of the beam using the method of integration.

[2 marks] (b) How would you compute the deflection of this beam in a design situation?





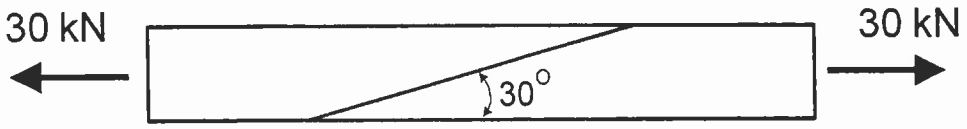
beam cross-section
(all dimensions in mm)

Question 6: The axial wood member 50 mm x 100 mm in cross section is subjected to an axial load equal to 30 kN. The member is made of two pieces that are glued together using a scarf joint with 30° angle to the horizontal axis as shown below.

[20 marks] Use the Mohr's circle solution (*not* the transformation equations) to determine the following:

(a) the stress components acting on the glued joint, showing your answer on a properly oriented element.

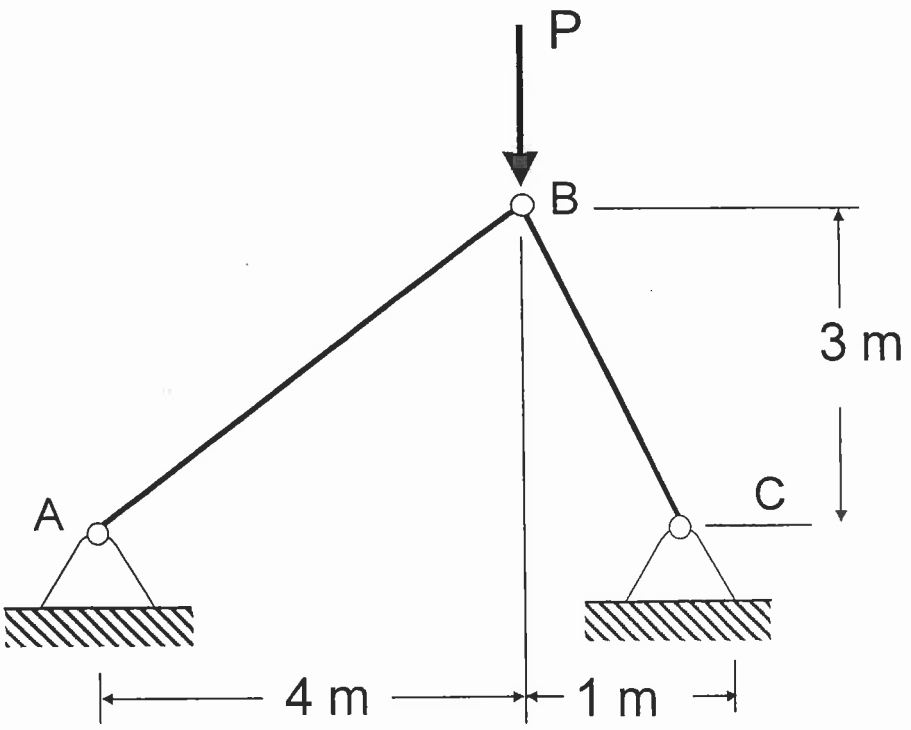
(b) the maximum in-plane shear stress of the member (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.



Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 7: [20 marks] Determine the largest load P that can be applied to the frame structure below given that members AB and BC are made of 100 mm outside diameter hollow steel rods with a 5 mm wall thickness. Both steel rods are pinned at their ends.

Consider in-plane buckling only for the compression members and use a factor of safety of 2 for the Euler buckling load. The steel used in the rods has an elastic modulus of E = 200 GPa and an allowable yield strength equal to 240 MPa.



Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$

Question 8: [20 marks] An inclined axial load of 2400 kN is applied to a bent element with the cross section as shown below. The element is made of steel with a yield stress of 310 MPa and elastic modulus of 200 GPa. Compute the normal stress distribution and shear stress distribution of the section at ground level (where the section is fixed). Show your answers on a sketch, and make sure to show maximum and minimum values of stress?