NATIONAL EXAMS May 2011
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates my use one of two approved Casio or Sharp calculators.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

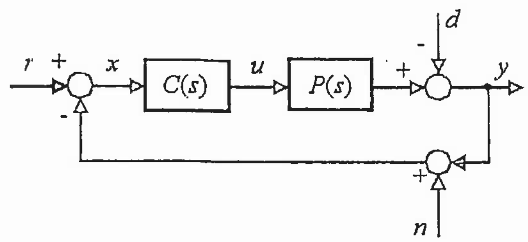
5. All questions are of equal value.

1. Consider the control system below with, $P(s) = \frac{1}{s^2 - 4}$ and $C(s) = \frac{K_1 s + K_0}{s + 20}$.

(a) Determine the range of K_1 and K_0 for closed loop stability.

(b) Determine K_1 and K_0 such that i) the absolute value of the steady state error is less than 0.1 when r is a unit step with $d = 0$ and $n = 0$, and ii) the closed loop system is stable.

(c) Determine the steady state value of $y(t)$ when $r(t) = 0$ and $d(t) = 0$ and $n(t) = 10\cos(5t)$.



2. Consider the open loop system,

$$\ddot{\phi}(t) = 10^{-3}\psi(t) - 10^{-2}u(t)$$
$$\dot{\psi}(t) = -10^{-1}\psi(t) + 10u(t)$$

(a) Determine a state space model for the open loop system assuming the state vector is given by, $x(t) = \begin{bmatrix}\phi(t) & \dot{\phi}(t) & \psi(t)\end{bmatrix}^T$, the control input, $u(t)$, and the output, $\phi(t)$.

(b) Determine the transfer function for the open loop system that relates $\phi(t)$ to $u(t)$.

(c) Assuming all of the states are available for feedback, specify a state feedback controller, if it exists, such that the closed loop poles are located at $s = -10, -1+j1, -1-j1$.

3. The discrete time model for a system has the form, $y(z) = P(z)u(z)$ where $P(z) = \frac{az^{-1}}{1 - bz^{-1}}$.

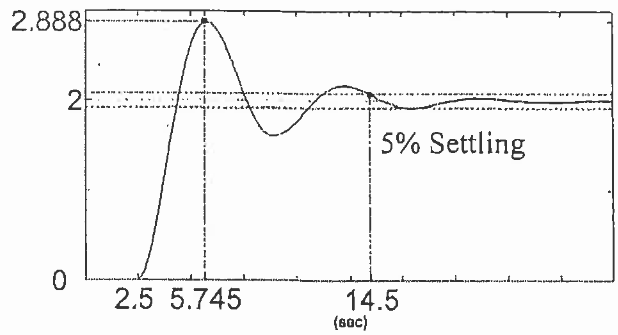
Measurements of $u(k)$ and $y(k)$ are taken at time instants, k, as listed in the table below.

(a) Find the least squares estimate for a and b.

(b) Assuming $y(k)$ is sampled uniformly from the output of a continuous time (linear time invariant) system and the sample $u(k)$ is applied uniformly to a ZOH that drives the input of the continuous time system, determine the transfer function of the continuous time system assuming the sample period is $h = 1$s.

k	$y(k)$	$u(k)$
0	9.0	0.00
1	2.0	0.25
2	2.5	0.50
3	4.5	-

4. A unit step is applied at the input of an open loop plant, $P(s)$, at time $t = 0$. The measured response is shown on the right.

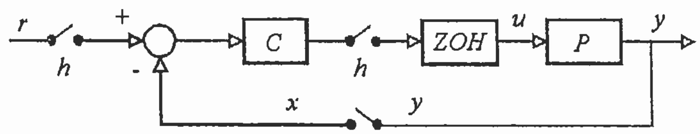


(a) Establish whether the response exhibits non-minimum phase behavior and/or a transportation delay.

(b) Determine the transfer function, $P(s)$.

(c) Justify whether the closed loop system is stable when the system is controlled with unity negative feedback.

5. Consider the sampled data system shown on the right. The input to the ZOH, the set-point, r, and the output, y, are uniformly sampled with a sample period of h. $C(z)$ and $P(s)$ are given by,



$$C(z) = \frac{c_1 z + c_0}{z - 1}, \quad P(s) = \frac{1}{s+1}$$

(a) Determine $C(z)$ such that the closed loop poles are all located at zero. You may solve as a function of h or you may assume $h = 0.4$s

(b) Determine the corresponding closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.

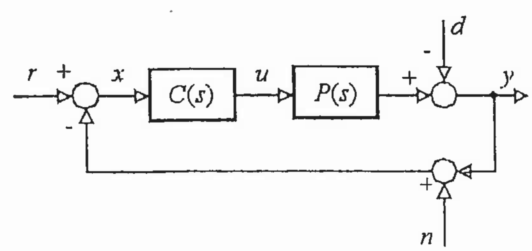
(c) Sketch the associated unit step response at $y(t)$.

6. Consider the feedback system below with, $C(s) = \dfrac{K(s+1)}{s}$, $P(s) = \dfrac{e^{-s/20}}{s}$.

(a) Determine the maximum value of $K = K_{max}$ for closed loop stability.

(b) For $K = K_{max}/2$ determine the gain crossover frequency and sketch the associated Bode plot, magnitude only.

(c) The system is stable and operating with constant inputs, $n(t) = 0$, $d(t) = d_0$, and $r(t) = r_0$. Determine the steady state tracking error, $e(t) = r(t) - y(t)$, as a function of K.



Inverse Laplace Transforms	
$F(s)$	**$f(t)$**
$\frac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\frac{C+jD}{s+\alpha+j\beta}+\frac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\frac{A}{(s+\alpha)^{n+1}}$	$\frac{At^n e^{-\alpha t}}{n!}$
$\frac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\frac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\frac{2t^n e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	**$f(nT)$**
$\frac{Kz}{z-a}$	Ka^n
$\frac{\left(C+jD\right)z}{z-re^{j\varphi}}+\frac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^n\left(C\cos n\varphi-D\sin n\varphi\right)$
$\frac{Kz}{\left(z-a\right)^r}$, $r=2,3...$	$\frac{Kn\left(n-1\right)...\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^n$

Table of Laplace and z-Transforms (h denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z\left(z-e^{-\alpha h}\cos\beta h\right)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$