National Exams May 2011

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1. a) Using the Routh-Hurwitz criterion, investigate the stability of the following characteristic equation:

$$S^5 + S^4 + 2S^3 + S^2 + S + K = 0$$

b) What is the unit step response of a system whose transfer function is given by

$$P(s) = \frac{(S + 2)}{(S + 0.5)\ (S + 4)}$$

c) A system is designed to give satisfactory performance when a particular amplifier gain K has the value 2. Determine how much this gain can vary before the system becomes unstable if the characteristic equation is:

$$S^3 + (4 + K)\,S^2 + 6\,S + 16 + 8\,K = 0$$

2.

a) Determine if the following characteristic equation represents a stable system:

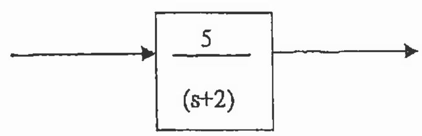
$$s^3 + 4s^2 + 8s + 12 = 0$$

b) The characteristic equation of a given system is:

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

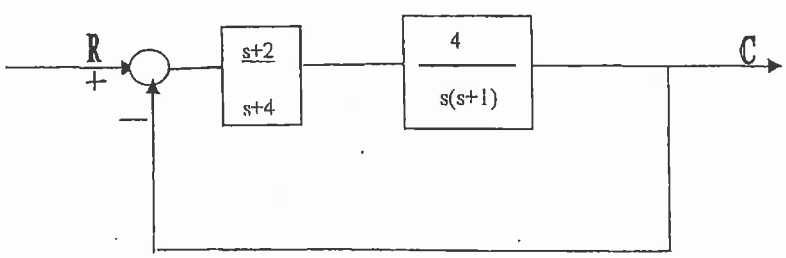
What restrictions must be placed upon the parameter K in order to insure that the system is stable?

c) The block diagram below depicts a first order lag system.



Derive an expression for the output if the input, (at time t = 0), is a unit ramp. How would you obtain the response to a unit impulse from the expression which you have derived?

3. For the stable system



a) Determine the system type.

b) Find the steady-state error for a unit step input, a unit ramp input, and a unit parabolic input.

4. a) The characteristic equation for a feedback control system is:

$$(s + 2)\,(s^2 + 4s + 8) + K = 0$$

Determine the range of values of K for which the system is stable.

b) The dynamics of a system are described by the differential equation:

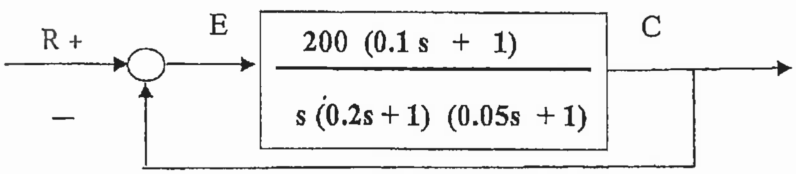
$$y(t) = \frac{10\,(2D + 1)}{(D + 2)\,(D + 5)}\, f(t)$$

Where $D = \dfrac{d}{dt}$

Use the Laplace transform method to determine the response y (t) when all initial conditions are zero and the forcing function f (t) is a unit step function.

5.

Given:



a) Draw the Bode diagram.

b) Is the system stable?

6. a) Draw the root locus for a unity feedback system with:

$$G(s) = \frac{K(s+1)}{s^2(s+9)}$$

Find the gain when all three roots are equal.

b) A control system for an ac induction motor has negative unity feedback with

$$G(s) = \frac{K(s^2+4s+8)}{s^2(s+4)}$$

It is desired that the dominant roots have an ξ equal to 0.5. Using the root locus, find the dominant roots and the corresponding K.

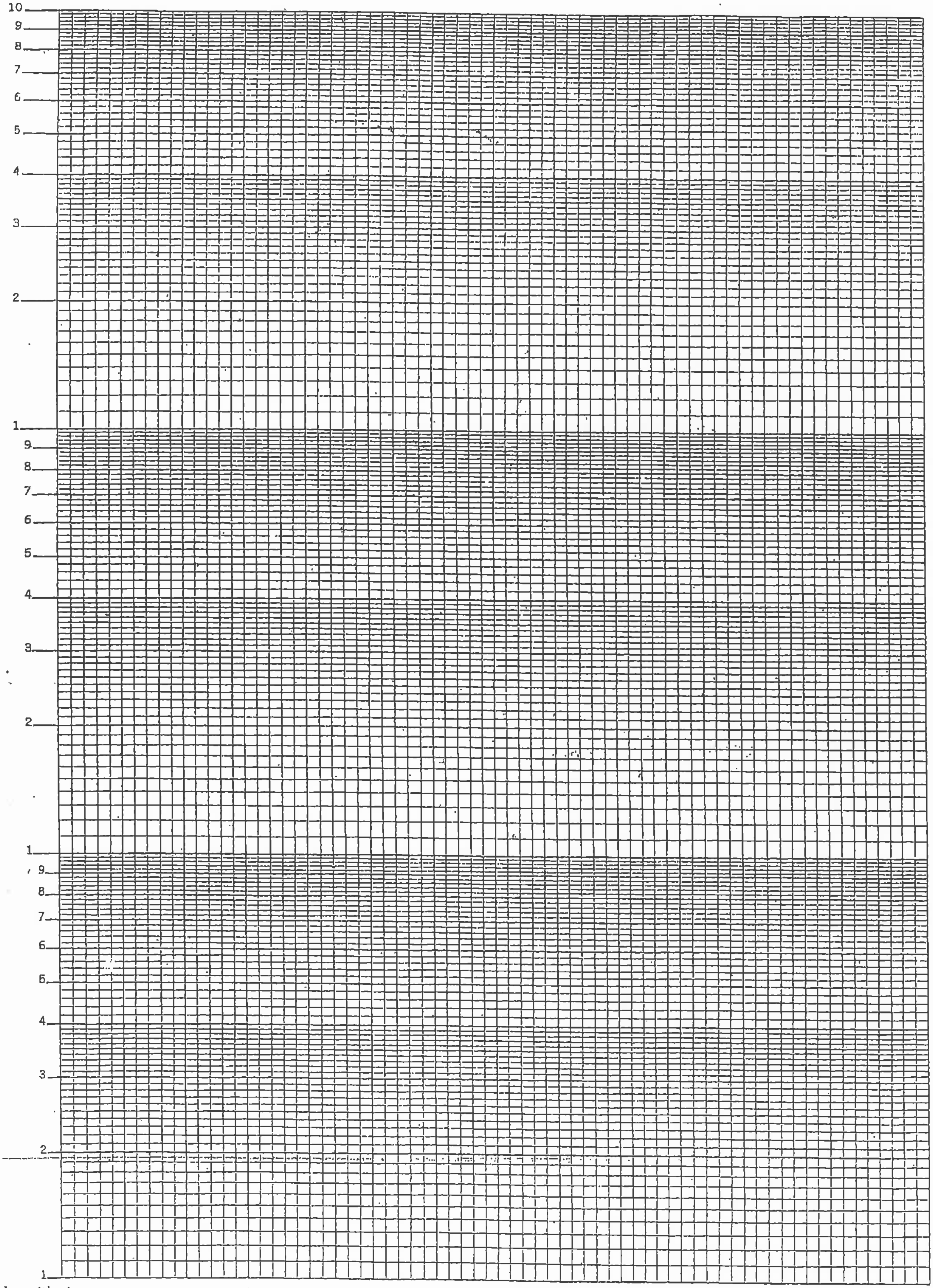
Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{2}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$

NATIONAL 12-183



Semi-Logarithmic