National Examinations May 2011

07-Mec-B10 – Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. **If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) that he/she has had made with the answer.**

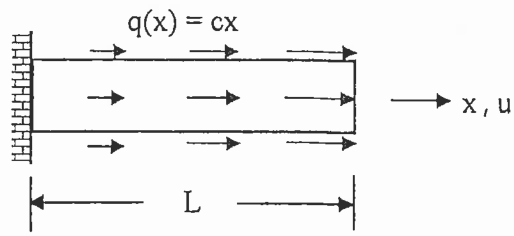
2. **The examination paper is open book and so candidates are permitted to make use of any textbooks, references or notes that they wish to use.**

3. **Any non-communicating calculator is permitted. A calculator that can handle small matrices will speed the solving of the problems. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**

4. Candidates are required to attempt five questions. **Solve all problems using finite element method.**

5. All questions carry the same value. Indicate which five questions are to be marked on the cover of the first examination workbook.

Question 1. [20 marks] A cantilevered bar is loaded by a linearly varying distributed load $q(x) = cx$ as shown in the figure - note that c is a constant. The cross-sectional area and length of the bar are denoted by A and L, respectively, and it is made of a material with Young's modulus of elasticity E. The system governing equation can be written as



$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\textit{subject to}: \quad u(x=0) = 0 \quad \textit{and} \quad EA\frac{du(x)}{dx}\bigg|_{x=L} = 0$$

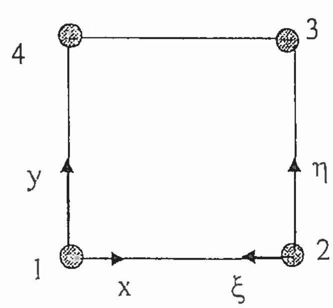
Use the Collocation method to determine an approximate cubic polynomial solution with evaluation points at $x = \frac{1}{2}L$ and $x = L$.

Question 2. [20 marks]

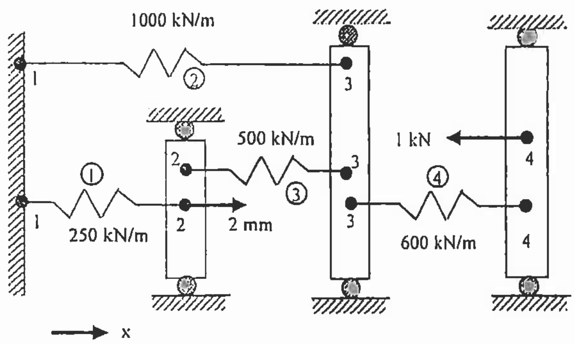
(a) [4 marks] Briefly explain the meaning of geometric isotropy in a sentence or two.
(b) [6 marks] State the **two** properties of a polynomial representation of the field variable variation in an element to ensure that the element has geometric isotropy.
(c) [10 marks] Consider the square element below for which the field variable u is interpolated in the Cartesian x,y coordinate axes attached at node 1 as

$$u(x, y) = C_1 + C_2x + C_3y + C_4xy$$

Assume that the length of each side of the element is L, use the ξ, η coordinate axes centred at node 2 to show that the element has geometric isotropy.

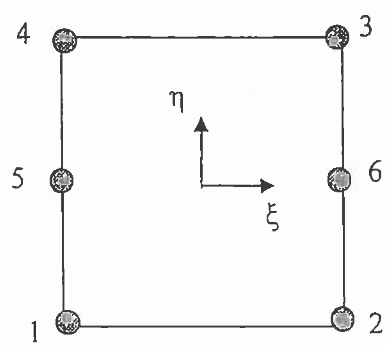


Question 3. [20 marks]

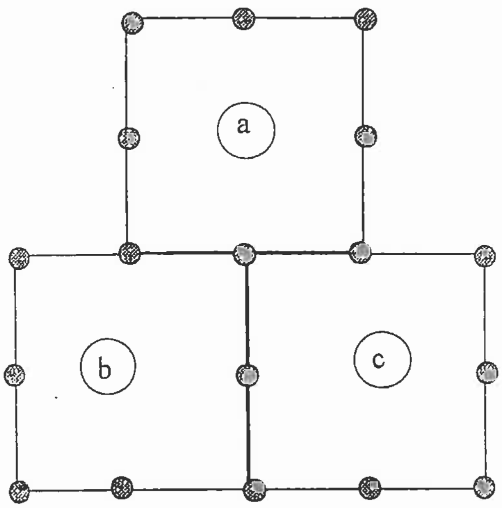


For the spring assemblage shown in the above figure,
(a) Write the total potential energy of the system.
(b) Apply the principle of minimum potential energy to your expression in part (a) to derive the system equilibrium equations.
(c) Write the system equilibrium equations in a matrix form.
(d) Determine the nodal displacements and reaction forces.
(e) Determine the forces in element #1 only.

Question 4. [20 marks] A six-noded transition element is shown in the figure below.



(a) Determine the shape functions.
(b) Explain whether the shape function $N_1(\xi,\eta)$ satisfies C^0 interelement continuity along the sides that contain node 1.
(c) Explain whether this element can connect to a nine-node biquadractic quadrilateral element along side 2-6-3 while satisfying compatibility.
(d) Explain the problem in the finite element mesh below involving 8-node serendipity elements, namely a, b, and c.



Question 5. [20 marks] A function $f(x,y) = x^2 y$ is defined over a rectangular doman $\Omega = \{\Re^+ : 0 \le x \le 4,\ 0 \le y \le 5\}$. Given the expression

$$g = \int_0^5 \int_0^4 x^2 y \, dx dy$$

and assume the variables x and y are interpolated as bilinear elements with the following shape functions:

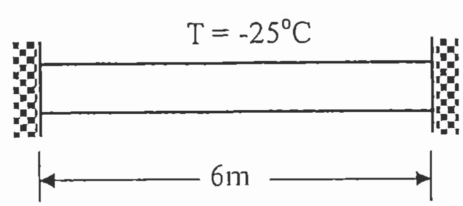
$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1+\xi)(1+\eta),\ N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

where $-1 \le \xi, \eta \le 1$ for the local coordinates ξ, η.

(a) Determine the value of g using the Gauss quadrature numerical integration method

(b) Compare your answer to the exact solution, giving reasons for any similarity or difference.

Question 6. [20 marks] A one-dimensional steel bar fixed at each end is shown in the figure. The properties of the bar are: Young's modulus $E = 210$ GPA, cross-sectional area $A = 1 \times 10^{-2}$ m^2, and coefficient of thermal expansion $\alpha = 12 \times 10^{-6}$ (mm/mm)/°C. If the bar is subjected to a uniform temperature drop of $T = 25$°C as shown, determine the reactions at the fixed ends and the stress in the bar.



Question 7. [20 marks]

(a) [3 marks] Briefly explain in a sentence or two the difference between basis function and shape function.
(b) [3 marks] Briefly explain in a sentence or two why finite element solutions improve with increasing number of elements or nodes.
(c) [4 marks] Briefly explain in a sentence or two why a discretization implemented using Bubnov-Galerkin method (i.e., Galerkin method) is identical to that obtained using Ritz method.
(d) [10 marks] An analyst notices a very slow convergence during a finite element analysis of a Timoshenko beam. On closer examination of the field variables, it is noticed that the transverse displacement was interpolated with a quadratic function and rotation by a linear function. What phenomenon is the analyst experiencing and how can the problem be resolved?