Professional Engineers Ontario

National Exams – May 2011
98-Civ-B9

Applications of the Finite Element Method

3 hours duration

Notes:

1. There are 4 pages in this examination.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made
3. This is a closed book examination.
4. Candidates may use one of the approved Casio or Sharp calculators.
5. **Answer only TWO (2) problems out of the three proposed.** The first two problems as they appear in the answer book will be marked.
6. Candidates are allowed to bring **ONE** aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae.
7. All problems are of equal value.

Problem 1: (If you select this problem, you must answer questions 1. through 10. below)

1. Explain the source of errors in finite element modelling.

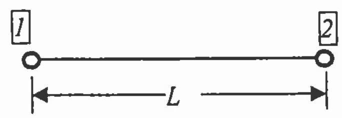
2. Explain why the displacement obtained at the nodes of a framed structure are exact. What about the displacement between the nodes?

3. Select the right answer to the following statement and explain your answer;
   "*The strain energy of an elastic structure calculated by the finite element method is:*"

   a. higher than the exact value
   b. equal to the exact value
   c. lower than the exact value

4. Give the characteristics of the shape functions needed for the Bernoulli's beam formulation.

5. Draw the approximate shape functions for the **beam element** shown in the following figure.



Element 1
Figure 1

6. Explain the relationship between the number of integration points and the order of a 2D elasticity finite element.

7. Explain when is it possible to reduce a 3D elasticity problem to a 2D model?

8. Explain why linear quadrilateral elements are not good candidates for flexural dominant two-dimensional elasticity problems?

9. Finite element analysis gives stresses in general coordinate directions in terms of σ_x, σ_y, etc. Discuss how you can interpret these results for ductile (e.g. mild steel) and brittle (e.g. concrete) materials.

10. What does the term "C^1 plate element" mean?

Problem 2

A symmetrical indeterminate truss subjected to a pair of forces is shown in Figure 2. All bars have the same rigidity *EA*. There is no connection between members (1,3) and (2,4).

Find the displacements at all joints and the axial force acting within each bar. Use the node numbering scheme shown in Figure 2.

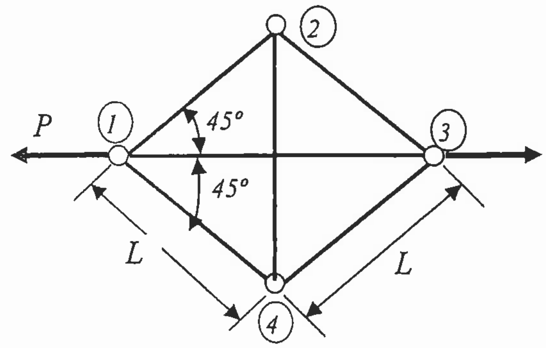
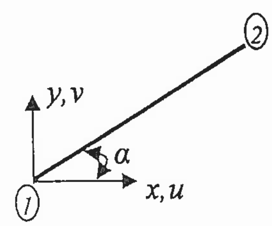


Figure 2

The stiffness matrix of a bar with orientation defined by an angle α with respect to the global reference system (x, y) is given by:

$$[k] = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \qquad C = \cos(\alpha), \quad S = \sin(\alpha)$$



The internal force in the bar is given by:

$$F_{(1,2)} = \frac{EA}{L}\begin{bmatrix} C & S \end{bmatrix}\begin{bmatrix} u_2 - u_1 \\ v_2 - v_1 \end{bmatrix}$$

where u_1, v_1 are the displacement of node 1 in the global reference
and u_2, v_2 are the displacement of node 2 in the global reference

Problem 3

Using two beam elements, calculate the reactions and draw the shear and moment diagrams for the structure shown below (Fig. 3). All members have the same rigidity. $E = 200\ GPa$, $I = 40 \times 10^6\ mm^4$.

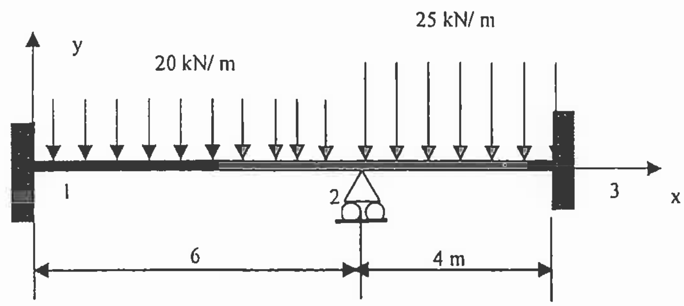


Figure. 3

The stiffness matrix of the beam element is shown below.

$$[k] = \frac{EI}{L}\begin{bmatrix} \frac{12}{L^2} & \frac{6}{L} & -\frac{12}{L^2} & \frac{6}{L} \\ \frac{6}{L} & 4 & -\frac{6}{L} & 2 \\ -\frac{12}{L^2} & -\frac{6}{L} & \frac{12}{L^2} & -\frac{6}{L} \\ \frac{6}{L} & 2 & -\frac{6}{L} & 4 \end{bmatrix}$$