National Exams December 2012

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

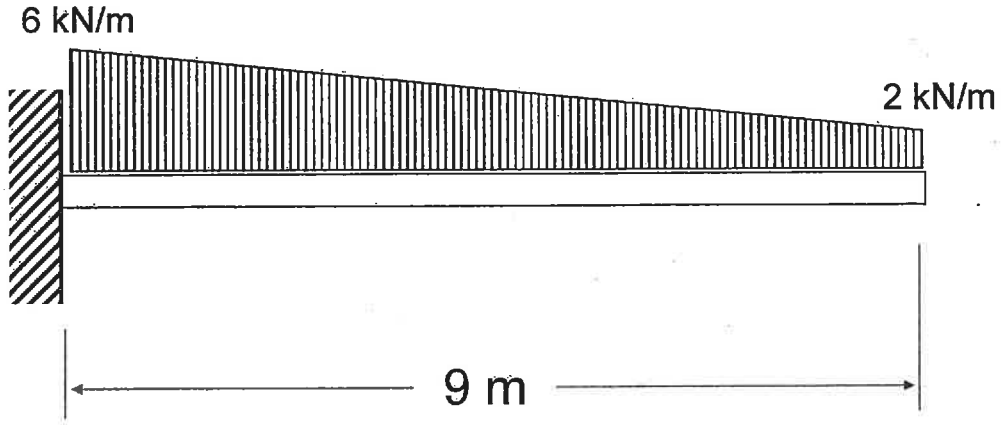
Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

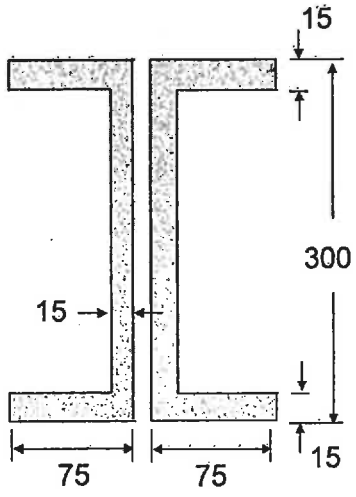
Question 1: [20 marks] A cantilevered beam (fixed at the left) with a 9 m span supports a distributed load as shown below. The beam is made up of two channel sections as shown in the cross-section, and is made from steel having an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel used in the beam equals 200 GPa.

(a) determine the maximum deflection of the beam using the method of integration

(b) determine the slope at the free end of the beam

[bonus] (c) indicate whether the maximum deflection computed is reasonable or not and give reasons for your answer





Beam Cross-Section
(all dimensions in mm)

No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

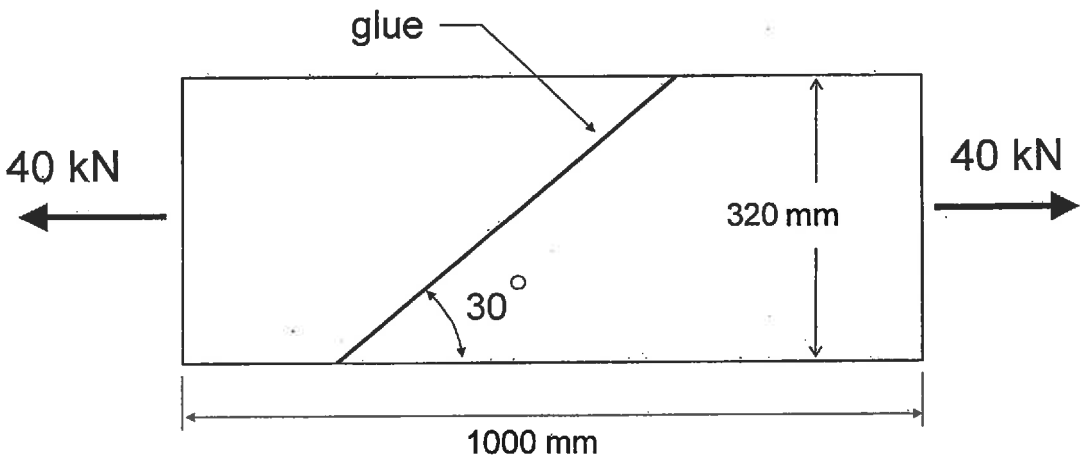
Question 2: Two wood plates 320 mm wide and 25 mm thick are glued together at an angle of 30 degrees to the horizontal axis as shown. The glued plate assembly resists an axial load of 40 kN.

[20 marks] Use the Mohr's circle solution (*not* the transformation equations) to determine the following:

(a) the normal and shear stress acting on the glued joint. Make sure to show your answer on a properly oriented element.

(b) the maximum in-plane shear stress (and associated normal stresses) acting on the wood plates. Once again, show your answer on a sketch of a properly oriented element.

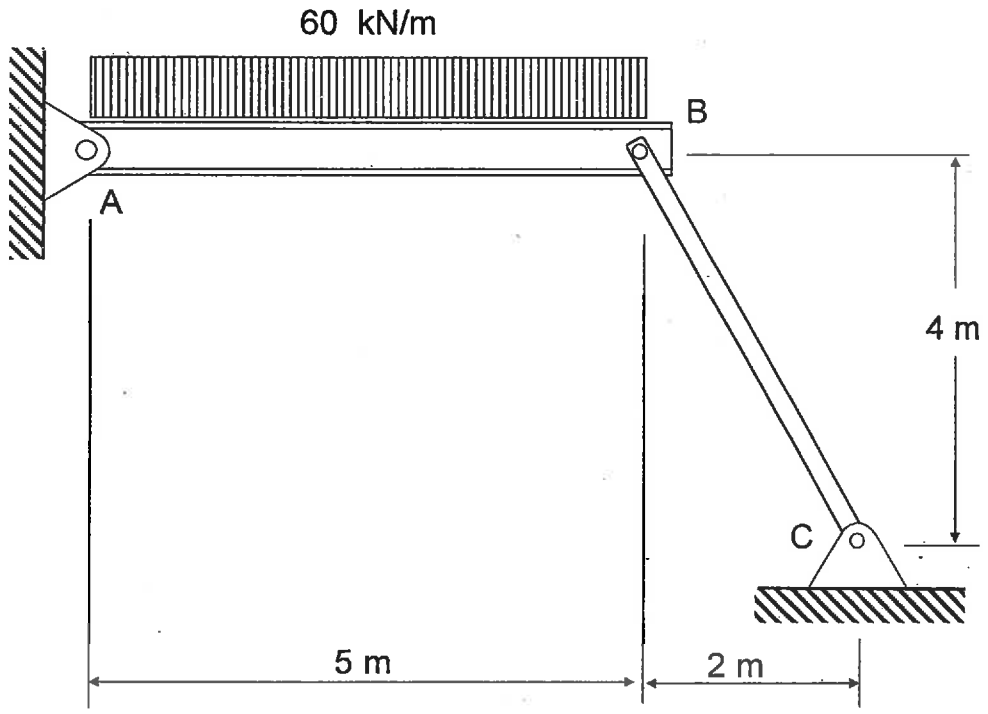
(c) briefly explain any alternative methods that you could use to solve this problem



Note that credit will **only** be given for a **solution using Mohr's circle.** This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 3: [20 marks] The horizontal beam AB is propped up by an inclined strut BC and supports a uniformly distributed load w = 60 kN/m. The strut is a 100 mm diameter steel rod pinned at both ends. The steel has a yield strength of 350 MPa and elastic modulus of 200 GPa.

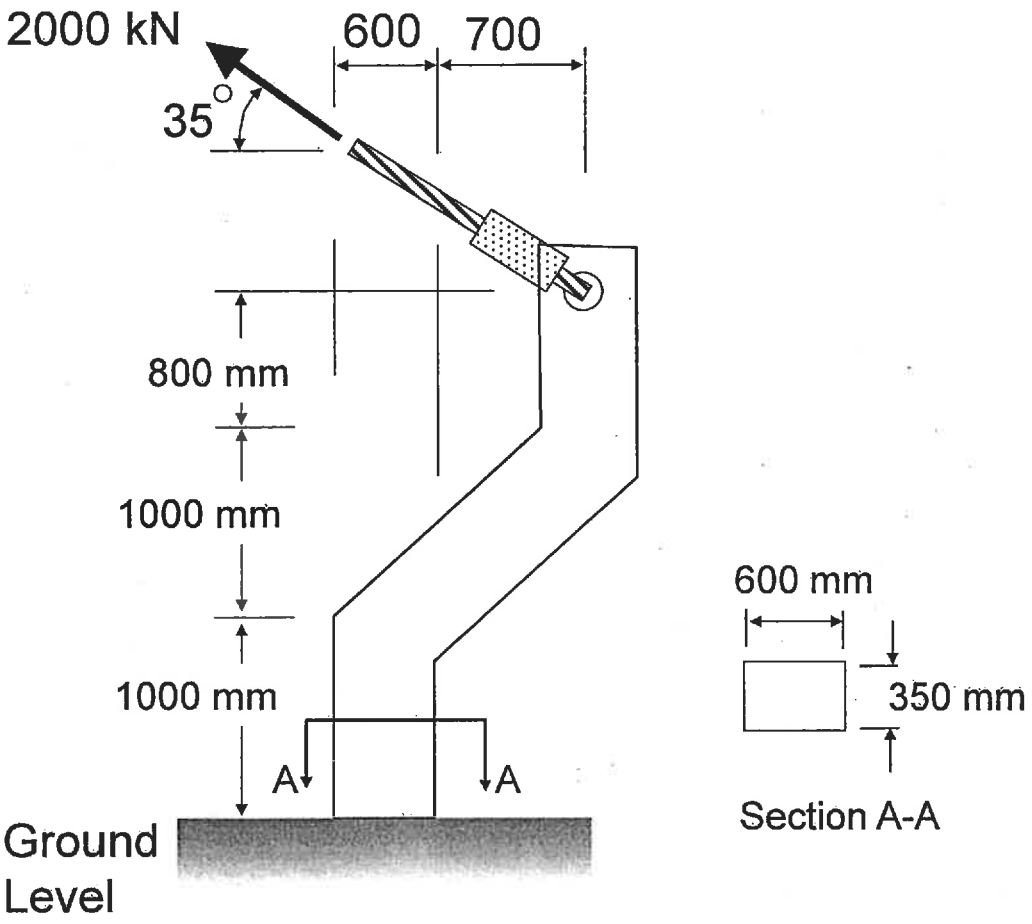
Determine the factor of safety against buckling of the inclined strut. Consider buckling in the plane of the structure only. Do not use a safety factor for yielding of the steel.



Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$

Question 4: [20 marks] A 2000 kN load (acting at 35 degrees to the horizontal) is applied to a bent element with the cross section as shown. The element is made of steel with a yield stress of 350 MPa and elastic modulus of 200 GPa.

Compute the maximum and minimum values of normal stress and shear stress acting on the section at ground level (where the section is fixed). Then show your answers with a sketch of the normal stress distribution and shear stress distribution acting across the section.

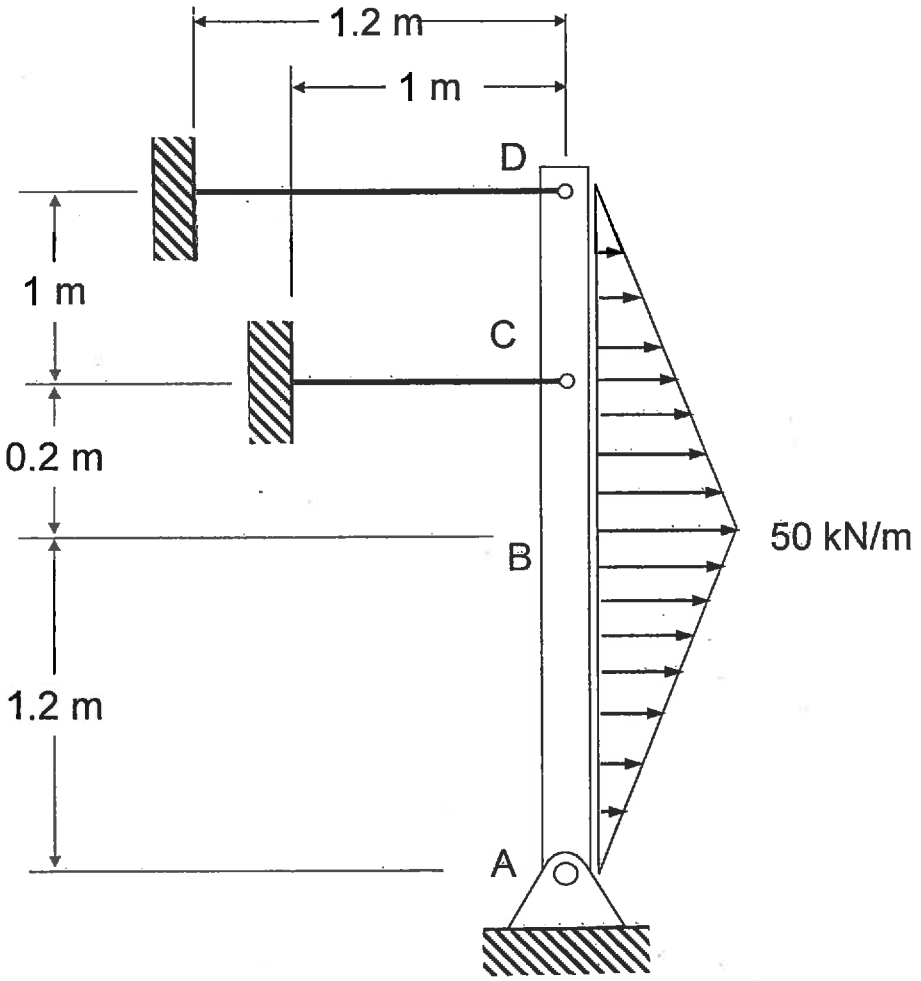


Question 5: A rigid vertical bar (ABCD) is supported by a 20 mm diameter pin at A and two 12 mm diameter cables located at points C and D. The cable at C has a length of 1 m while the cable at D has a length of 1.2 m. Both cables are made of high strength steel with a yield strength of 800 MPa and elastic modulus of 200 GPa. The rigid bar is loaded with a triangularly distributed load having a maximum intensity of 50 kN/m half way up the bar.

[12 marks] (a) find the forces developed in each cable

[4 marks] (b) find the corresponding horizontal displacement at point B (halfway up the bar)

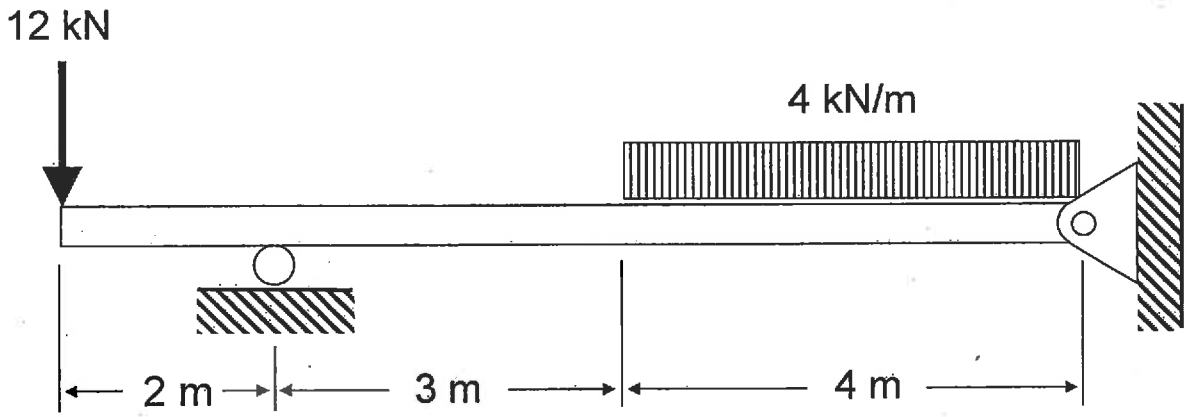
[4marks] (c) find the shear stress in the pin at A given that the pin is loaded in double shear



Question 6: [20 marks] A simply supported beam with an overhang is subjected to a uniformly distributed load over part of the span in addition to a concentrated load acting at the end of the overhang.

Determine the shear and moment throughout the beam as functions of x. This means that you need to give formula(s) for **V(x)** and **M(x)** along the length of the beam.

Next construct the shear force and bending moment diagrams. Remember to label points of maximum positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.



Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

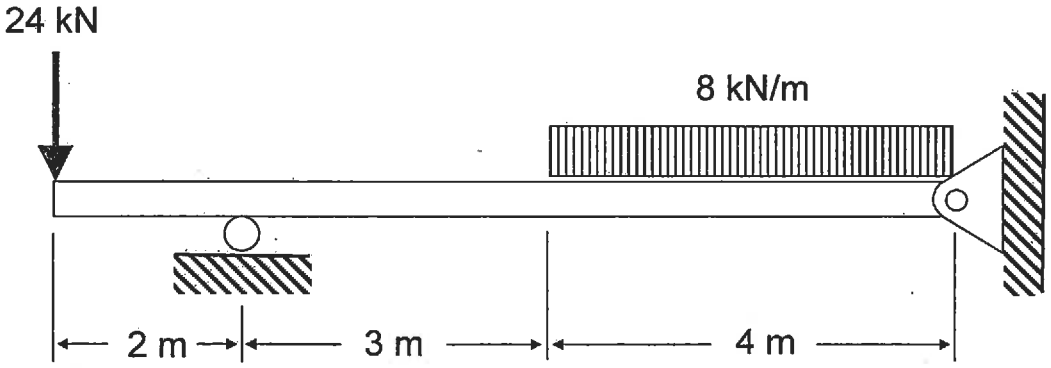
No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

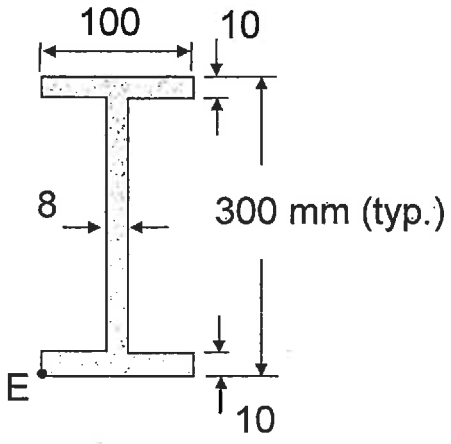
Question 7: [20 marks] A simply supported beam with an overhang is subjected to a uniformly distributed load over part of the span in addition to a concentrated load acting at the end of the overhang. The beam is a wide flange section (see below) using steel with an allowable normal stress of 240 MPa and an allowable shear stress of 75 MPa. The elastic modulus of the steel is 200 GPa.

Do the following:

(a) determine the maximum (absolute) normal stress and maximum shear stress in the beam.

(b) determine the shear stress at the tip of the flange (point E) for the beam section at the pinned support. To receive marks you must give reasons to justify your answer.





beam cross-section
(all dimensions in mm)

Question 8: A circular stepped shaft is fixed at the left end (point A) and subjected to two torques (points C and D) acting as shown below. Part of the shaft (AB) is hollow and the entire shaft is made of aluminum with G = 25 GPa and a yield stress of 200 MPa. Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

[12 marks] (a) determine the maximum shear stress in the stepped shaft and sketch the corresponding variation of shear stress along the shaft radius at this point.

[6 marks] (b) find the angle of twist at the end of the shaft (point D) and give your answer in degrees.

[2 marks] (c) A shoulder fillet is used at the step in the shaft (point C) to reduce the stress concentration factor K to a value of 2.2. Is this sufficient to avoid failure in the shaft?