National Exams

07-Elec-B1, Digital Signal Processing

December 2012

3 Hours Duration

NOTES:

1. **If doubt** exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM, One of two calculators, any Casio or Sharp approved models.
3. FOUR(4) questions constitute a complete paper. The first four questions as they appear in the answer book will be marked.
4. All questions are of equal value.
5. Clarity and organization of the answer are important.
6. One aid sheet is allowed 8.5" x 11" hand written on both sides containing notes & formulae.

1. **(25 marks total)** Determine the output $y[n]$ (total solution) for $n \geq 0$ of a linear time invariant (LTI) discrete system characterized by the difference equation given below to the following input signals $x[n]$:

$$y[n] + y[n-1] - 6y[n-2] = x[n]$$

   (a) **(13 marks)** $x[n] = 8u[n]$ (unit step function), with initial conditions $y[-1] = 1$ and $y[-2] = -1$

   (b) **(12 marks)** $x[n] = 2^n u[n]$ with the same initial conditions above.

2. **(25 marks total)** The following two parts (a) and (b) are independent.

   (a) **(15 marks)** Let $y[n]$ be the sequence obtained by a linear convolution of two causal finite-length sequences $h[n]$ and $x[n]$. For the following pair of $y[n]$ and $h[n]$, determine $x[n]$. The first sample in each sequence is at time index $n = 0$.

$$\{y[n]\} = \{6, 11, -13, 16, 1, 9, 2, 8\}, \{h[n]\} = \{2, 5, -1, 4\}$$

   (b) **(10 marks)** Determine the even and odd parts of the sequence $x[n]$ given below.

$$\{x[n]\} = \{-4, 5, 1, -2, -3, 0, 2\}, -3 \leq n \leq 3$$

3. **(25 marks total)** The following two parts (a) and (b) are independent.

   (a) **(12 marks)** The frequency response $H(e^{j\omega})$ of a length-4 FIR filter with impulse response has the following specific values:

   $$H(e^{j0}) = 2, \qquad H(e^{j\pi}) = 0, \qquad H(e^{j\pi/2}) = 7 - j3.$$

   Determine its impulse response $h[n]$.

   (b) **(13 marks)** For the following LTI causal digital filter with impulse response given below, find the frequency response $H(e^{j\omega})$ and sketch the magnitude response of the filter.

   $$h[n] = 0.3\delta[n] - \delta[n-1] + 0.3\delta[n-2]$$

4. **(25 marks total)** A causal LTI system has the system transfer function

   $$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{(1 + \frac{1}{2}z^{-1})(1 - z^{-1})}.$$

   (a) **(10 marks)** Find the impulse response of the system, $h[n]$.

   (b) **(15 marks)** Find the output of this system, $y[n]$, for the input

   $$x[n] = e^{j(\frac{\pi}{2})n}.$$

5: **(25 marks total)** Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = \cos(\frac{\pi n}{2}), \quad n = 0, 1, 2, 3, \qquad h[n] = 2^n, \quad n = 0, 1, 2, 3.$$

(a) **(5 marks)** Calculate the four-point DFT $X[k]$.

(b) **(5 marks)** Calculate the four-point DFT $H[k]$.

(c) **(7 marks)** Calculate $y[n] = x[n] ④ h[n]$ by doing the circular convolution directly.

(d) **(8 marks)** Calculate $y[n]$ of part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.