Professional Engineers of Ontario

Annual Examinations – December 2012

07-Elec-B3
Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. One of two calculators is permitted any Casio or Sharp approved models.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(25 marks) 1. This question considers link budgeting.

(10 marks) a. Consider a wireless system with transmitter power of 1 W, antenna gains of 6 dB, receiver losses of 3 dB, receiver noise figure of -168 dBm/Hz, a bandwidth of 10 MHz, and a fading margin requirement of 9 dB. Aside from path losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 3 dB, what is the maximum allowed path loss (in dB)?

(10 marks) b. Using a free-space path loss of $20 \log_{10}(4\pi df/c)$, where d represents the distance from transmitter to receiver, f represents the carrier frequency, and c represents the speed of light ($c = 3.0\times10^8$ m/s), and assuming a carrier frequency of 1 GHz, find the maximum distance between transmitter and receiver given the system in part a.

(5 marks) c. Discuss the role of the path loss exponent in modifying the free-space path loss in part b.

(25 marks) 2. This question concerns the use of spread spectrum modulation.

(10 marks) a. Explain the operation of direct sequence spread spectrum, including signal modulation and detection. In what sense is this technique "spread spectrum"?

(10 marks) b. Explain the operation of frequency hopping spread spectrum, including signal modulation and detection. In what sense is this technique "spread spectrum"?

(5 marks) c. As the number of users increases, how does the performance of a spread spectrum system change? Compare this to the performance of a TDMA system with a fixed number of slots.

(25 marks) 3. This question concerns error-control coding.

(10 marks) a. Consider a binary error-correcting code with a minimum Hamming distance of *d*. At most how many errors can be corrected, and at most how many errors can be detected? For both detection and correction, fully explain your answer.

(5 marks) b. Consider a binary Hamming code with the following generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(10 marks) Find the parity check matrix of this Hamming code.

c. Using an example, show how this code can correct a single error. Show all work.

(25 marks) 4. This question concerns signal modulation and detection.

(5 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_0(t) = \begin{cases} \sin(2\pi t/T), & 0 \le t \le T; \\ 0 & \text{elsewhere} \end{cases}$$

and $s_1(t) = -s_0(t)$. Sketch the two signals, and sketch the impulse response of the matched filter $m(t)$, assuming the filter is matched to $s_0(t)$, and assuming the filter output is sampled at time T.

(5 marks) b. In the absence of noise, what is the matched filter output at time T, if $s_0(t)$ is sent? Trig identity if you need it: $\sin^2 x = (1 - \cos 2x)/2$

(5 marks) c. At the sampling instant (time T), the matched filter output is corrupted by additive Gaussian noise with zero mean and variance σ^2. Give the optimal decision rule assuming that 0 and 1 are equiprobable.

(10 marks) d. Given that

$$\frac{1}{2}\text{erfc}\left(\frac{t-\mu}{\sqrt{2\sigma^2}}\right) = \int_t^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

and given your decision rule from part c, express the probability of error *given than a 0 was sent* in terms of erfc.

(25 marks) 5. This question concerns sampling and D/A conversion.

(5 marks) a. An NTSC video signal has a bandwidth of 6 MHz. Using the Nyquist sampling criterion, what is the minimum sampling frequency that is needed to represent this signal exactly?

(10 marks) b. Briefly explain pulse code modulation (PCM). If PCM is used to encode the signal from part a with 16 bits per sample, what is the required data rate to represent the signal? (If you didn't get an answer for part a, assume a value.)

(5 marks) c. Suppose 16-bit PCM is used to sample a signal restricted between -8 V and +8 V. What is the maximum quantization error?

(5 marks) d. To obtain a signal of the same quality, modern video recording standards would use a much lower data rate than your answer from part b. Briefly explain why.

Marking Scheme

1. 25 marks
    a. 10 marks
    b. 10 marks
    c. 5 marks
2. 25 marks
    a. 10 marks
    b. 10 marks
    c. 5 marks
3. 25 marks
    a. 10 marks
    b. 5 marks
    c. 10 marks
4. 25 marks
    a. 5 marks
    b. 5 marks
    c. 5 marks
    d. 10 marks
5. 25 marks
    a. 5 marks
    b. 10 marks
    c. 5 marks
    d. 5 marks