National Exams December 2012

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

<u>NOTES:</u>

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **<u>or</u>** Sharp approved calculator. This is a **<u>closed book</u>** exam. No aids other than semi-log graph papers are permitted.

3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1. A unity feedback closed-loop system contains an open-loop transfer function

$$G(s) = \frac{9}{s(s+5)}$$

A factor $D(s) = Ks + 1$ is cascaded with $G(s)$, where K is an adjustable gain, as shown in Fig. 1.

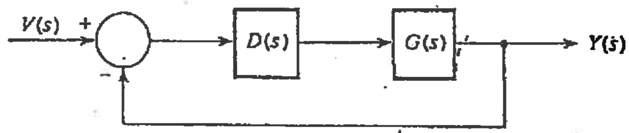

Fig. 1.

(a) Find the value of K necessary to provide a critically damped closed-loop system.

(b) Find the range of values of K which will provide acceptable damping for the closed-loop system.

(c) What is the magnitude of overshoot which follows a unit step input, when $K = 0.1$?

2.a. Find the steady-state parabolic error for the unity feedback closed-loop system whose open-loop transfer function is

$$G(s) = \frac{2.5}{s^2(2s+1)}$$

b. Find the steady-state step and ramp errors for the unity feedback closed-loop system whose open-loop transfer function is

$$G(s) = \frac{s+4}{s(s+5)}$$

3. Determine whether or not the open-loop system described by the transfer function $G(s)$ is stable, where

$$G(s) = \frac{1}{(s+3)(s-1)(s+2)}$$

A negative feedback loop is applied with transfer function $H(s) = K_1$, in which K_1 is a scalar gain value. Find the closed-loop transfer function and find the ranges of values for K_1 over which the closed-loop system is stable.

4. Determine whether or not the systems with the transfer functions given are stable.

(a) $\dfrac{s+1}{s^5 + 3s^4 + 4s^3 + 4s^2 + 3s + 1}$

(b) $\dfrac{1}{s^5 + 2s^4 + 6s^3 + 10s^2 + 8s + 12}$

(c) $\dfrac{1}{s^5 + 2s^4 + 3s^3 + 6s^2 + 10s + 15}$

5. A unity feedback system has an open-loop transfer function

$$G(s)=\frac{2.5K(s+2)}{(s-1)(s+1)}$$

Draw the root locus diagram for this system and find the range of values for K over which the closed-loop system is stable.

6. The open-loop transfer function for a particular system is

$$G(s)=\frac{30(s+0.5)}{s^2(s+5)(s+3)}$$

For a unity feedback system, find the gain and phase margins by means of a Bode plot, and ascertain the damping ratio.

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha te^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$

Semi-Logarithmic
4 Cycles x 10 to the inch

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