National Exams December 2012

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. Answer any 4 of the 5 questions.
4. Weighting: Each question is equally weighted.

Answer any **4** of the following 5 questions.

Question 1: A centrifugal pump, having four stages in parallel, delivers 218 litres/s of liquid against a head of 26 m, the diameter of the impellers being 229 mm and the speed 1,700 rpm. A pump is to be made up with a number of identical stages in series of similar construction to those in the first pump. The new pump is to run at 1,250 rpm, delivering 282 litres/s against a head of 265 m. Find the diameter of the impellers and the number of stages required. (Hint: It is the stages, not the machine which are similar). Use: $g = 9.81$ m/s^2.

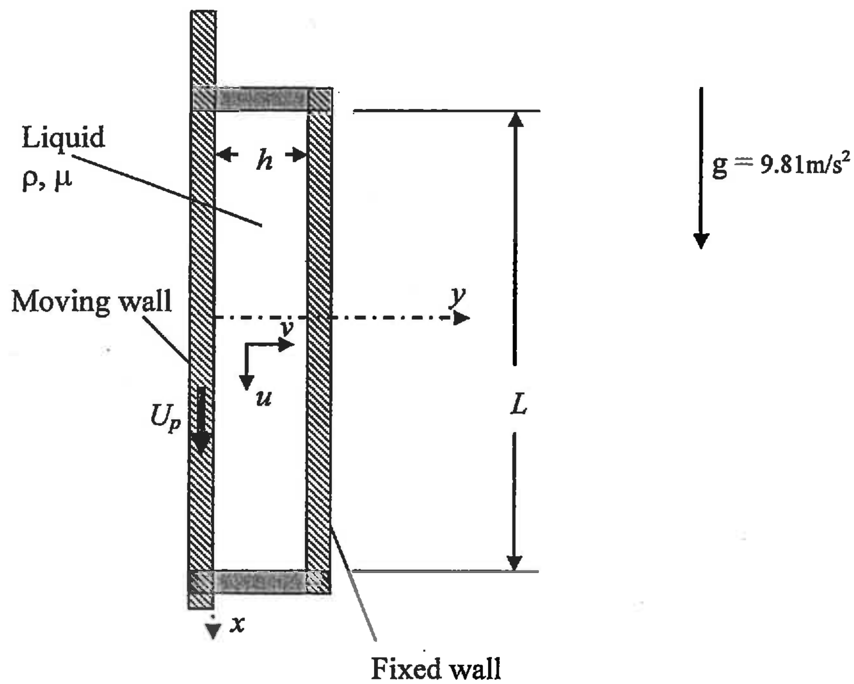
Question 2: Air at a density of 7 kg/m^3 and a temperature of 365K at the inlet flows through a convergent-divergent nozzle at a rate of 0.5 kg/s. At the inlet, the cross-sectional area is 10 cm^2. If the density at the exit is 4.8 kg/m^3 and the temperature is 350K, what is the cross-sectional area at the plane where the shock is located? Also, determine the Mach number and static pressure at the exit and the Mach number directly upstream of the shock. Assume the flow to be frictionless and adiabatic.
For air: $R = 287$ J/kg-K; $\gamma = 1.4$.

Question 3: Hydrogen is pumped through a 2m inner diameter pipeline from Edmonton to Calgary (300km). The gas leaves the first booster station at a mass flow rate of 74.3 kg/s and static conditions of -30°C and 1MPa. Determine the maximum distance between booster stations. Given that each station has a compressor and cooler to maintain the same outlet conditions as the first booster station and that the total-to-total efficiency of the compressor is 0.85, determine the power consumption for each compressor and the heat rejected by each cooler. If the power to drive the compressors costs \$0.12/kW-hr, what is the operating cost per kg of delivered hydrogen? At what static pressure and temperature is the hydrogen delivered?

The friction factor for the pipeline is given as 0.03 and the system may be considered well insulated. For hydrogen use: $\gamma = 1.4$, $R = 4.124$ kJ/kg-K, $Cp = 14.35$ kJ/kg-K.

Question 4: Consider the flow of an incompressible, Newtonian liquid (of density ρ and dynamic viscosity μ) in a thin vertical enclosure of width *h* with one wall moving at a constant rate U_p, as shown schematically below. The enclosure is long ($L/h >> 1$) such that the flow may be assumed fully developed. You can also assume that the span (*b* into the page) is large ($b/h >>1$). It is further known that the the liquid wets the solid (non-porous) walls and that the flow is not changing in time.

a) State the simplifying assumptions.
b) What are the boundary conditions?
c) Show that $v = 0$ everywhere.
d) From the *x*-momentum equation, determine the *u*-velocity profile.
e) Noting that an enclosure is a closed system (*i.e.* no net flow rate), determine the resulting pressure gradient in terms of ρ, μ, U_p, *h* and g.
f) What is the magnitude and direction of the shear stress at the walls?



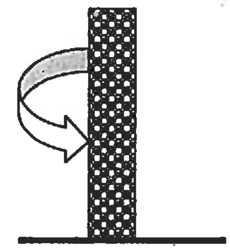
Schematic representation of thin enclosure filled with liquid and one wall moving at a constant rate.

<u>Question 5:</u> A water processing plant has as an intermediate stage of processing a large mixing pool in which bacteria are mixed to the waste components in the water. The pool is stirred slowly through a vertical stirrer located at $x = a$ and $y = b$. The stirrer can be modelled as a two-dimensional vortex of strength Γ. Since the pool is very large ($L/a >> 1$, $L/b >> 1$), it can be assumed that, as a first approximation, the resulting flow patterns are those for a vortex placed in an unbound corner ($L \rightarrow \infty$).

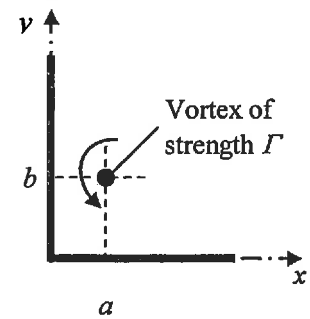
a) Show the stream-function for this flow field and show that the x and y axes satisfy the necessary conditions to represent a wall.

b) Determine the velocity distribution along the walls. Where is (are) the stagnation point(s)?

c) What is the pressure at the point $x = a$, $y = 0$ expressed in terms of Γ, a, b, the fluid density ρ and the stagnation pressure P_o ?



Side view of stirrer



Top view of pool with stirrer

Aid Sheets

Compressible Flow:

Adiabatic flow: $$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Isentropic flow: $$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \quad ; \quad \frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

$$\dot{m} = \rho U A = \sqrt{\frac{\gamma}{R}} \cdot \frac{P_o}{\sqrt{T_o}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma + 1}{2(\gamma - 1)}} A$$

Shock Relations: $$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^{\,2}}{1 + \gamma M_2^{\,2}} = \frac{2\gamma M_1^{\,2}}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \qquad M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

Conservation Equations for Cartesian Coordinate system

Continuity Equation:

$$\frac{D\rho}{Dt} + \rho\left(\nabla \bullet \vec{U}\right) = 0$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} \quad \text{and} \quad \nabla \bullet \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Linear Momentum:

***x*-direction:** $$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + \rho g_x + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$

***y*-direction:** $$\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + \rho g_y + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}$$

***z*-direction:** $$\rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial w}{\partial y} + \rho w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \rho g_z + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu\nabla \bullet \vec{U} \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu\nabla \bullet \vec{U} \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \qquad \tau_{zz} = 2\mu\frac{\partial w}{\partial z} - \frac{2}{3}\mu\nabla \bullet \vec{U}$$

Potential Flow

Stream functions:

Uniform Flow: $\Psi = U_o y = U_o r \sin\theta$

Source Flow: $\Psi = \frac{m}{2\pi}\tan^{-1}\left(\frac{y-y_o}{x-x_o}\right) = \frac{m}{2\pi}\theta$

Vortex Flow: $\Psi = -\frac{\Gamma}{4\pi}\ln\left[(x-x_o)^2+(y-y_o)^2\right] = -\frac{\Gamma}{2\pi}\ln r$

Doublet Flow: $\Psi = -\frac{\lambda(y-y_o)}{(x-x_o)^2+(y-y_o)^2} = -\lambda\frac{\sin(\theta)}{r}$

Potential functions:

Uniform Flow: $\Phi = U_o x = U_o r\cos\theta$

Source Flow: $\Phi = \frac{m}{4\pi}\ln\left[(x-x_o)^2+(y-y_o)^2\right] = \frac{m}{2\pi}\ln r$

Vortex Flow: $\Phi = \frac{\Gamma}{2\pi}\tan^{-1}\left(\frac{y-y_o}{x-x_o}\right) = \frac{\Gamma}{2\pi}\theta$

Doublet Flow: $\Phi = \frac{\lambda(x-x_o)}{(x-x_o)^2+(y-y_o)^2} = \lambda\frac{\cos(\theta)}{r}$

Velocity relationships:

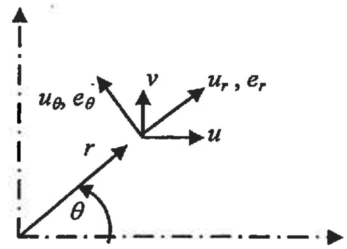
$$u = \frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y} \qquad v = \frac{\partial\Phi}{\partial y} = -\frac{\partial\Psi}{\partial x}$$

$$u_r = \frac{\partial\Phi}{\partial r} = \frac{1}{r}\frac{\partial\Psi}{\partial\theta} \qquad u_\theta = \frac{1}{r}\frac{\partial\Phi}{\partial\theta} = -\frac{\partial\Psi}{\partial r}$$

Transformation between Coordinate System

$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$

$x = r\cos\theta$ $y = r\sin\theta$



$\vec{e}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$ $\vec{e}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}$

$u_r = u\cos\theta + v\sin\theta$ $u_\theta = -u\sin\theta + v\cos\theta$

$u = u_r\cos\theta - u_\theta\sin\theta$ $v = u_r\sin\theta + u_\theta\cos\theta$