National Exams December 2012

98-Phys-B5, Control

3 hours duration

NOTES:

1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a single-sided, handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining five. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **PLEASE WRITE ANSWERS DIRECTLY IN THIS EXAM PAPER – DO NOT USE EXAM BOOKS.** If necessary, you may write on the backside of the pages as long as you write the final answers in the space indicated, and point to where the full calculations are.

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING FIVE QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
TOTAL:	100	

A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{s^2}$	$t \cdot 1(t)$
$\frac{1}{s^k}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$t \cdot e^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin(at) \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos(at) \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos(bt) \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin(bt) \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1-e^{-at} \cdot \left(\cos bt + \frac{a}{b}\sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n \sqrt{1-\xi^2}\, t) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$	$\left(1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cdot \sin(\omega_n \sqrt{1-\xi^2}\, t + \cos^{-1}\xi)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s)-f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Question 1 (Compulsory)

Proportional + Integral Controller Design in s-Domain, Dominant Poles Model, Step Response Specifications.

Consider a unit feedback closed loop control system, as shown in Figure Q1.1.

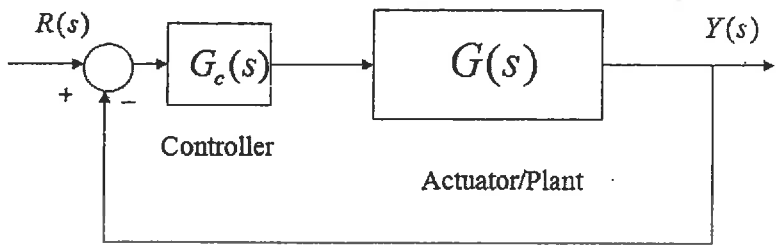

Figure Q1.1

The system is to operate under Proportional + Integral Control. The PI Controller transfer function is shown below:

$$G_c(s) = \left(\frac{K_i}{s} + K_p\right)$$

The process transfer function G(s) is as follows:

$$G(s) = \frac{4}{(s+10)(s+15)}$$

The closed loop performance requirements are:

- The Steady State Error for the unit step input for the compensated closed loop system is zero.
- Percent Overshoot is approximately 10%.
- Settling time within ±2% of the steady state, $T_{settle(\pm 2\%)}$, is approximately 2 seconds.

1) **(5 marks)** First, assume a closed loop model of this third order system to be of the following form:

$$G(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\cdot\frac{a}{s+a}$$

Next, assume that this model has a dominant pair of conjugate complex poles, meaning that the real closed loop pole, at $-a$, is placed so far away to the left of the complex plane that it has no significant effect on the system step response. Given this assumption, calculate the appropriate parameters K_{dc}, ζ and ω_n such that the closed loop specifications would be met. Place your results in Table Q1.1.

2) **(5 marks)** Derive the closed loop transfer function of the compensated system in terms of the PI Controller Gains, K_p and K_i.

3) **(5 marks)** Compare the characteristic equations of the assumed model and of the closed loop transfer function, and derive the values for the PI Controller Gains, K_p and K_i, as well as the real closed loop pole, a. Place your results in Table Q1.1.

4) **(5 marks)** Check if the assumption made in Item 1 is true, i.e. that the effect of the third closed loop pole is negligible. When can such an assumption be made? Is this condition met here? If it is not met, what is the expected effect of the real pole on the closed loop system response? Figure Q1.2 shows the response of PI Controller, compared with the response of a dominant poles model. Why are the two responses different? What factors are at play here? Comment briefly in the lined space provided.

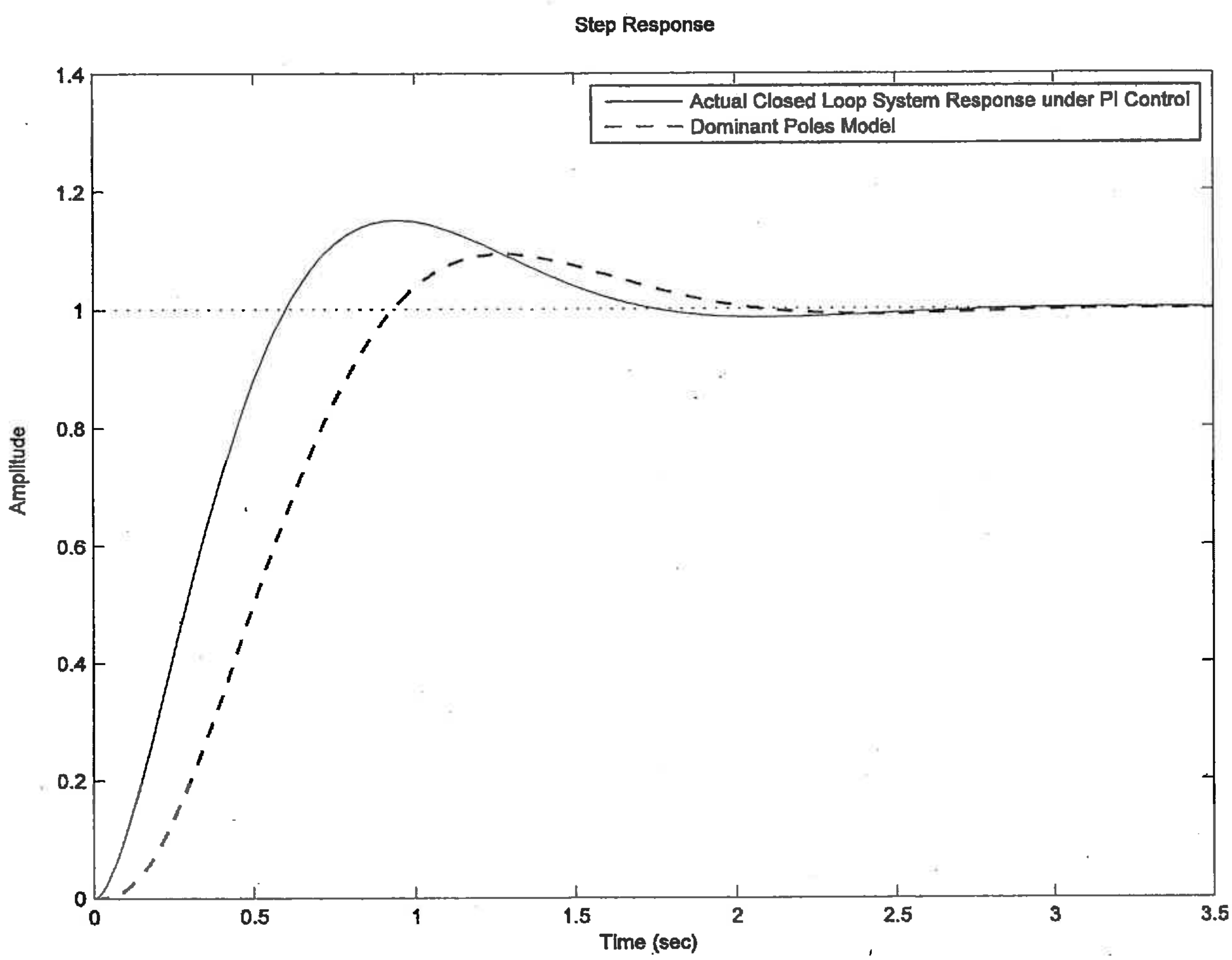

Figure Q1.2

Question 1 Continued

Question 1 Continued

Question 1 Continued

Table Q1.1

The closed loop system model with dominant poles will have the following parameters of its conjugate pair:		
$K_{dc} =$	$\zeta =$	$\omega_n =$
Third order characteristic equation has the following general form: $s^3 + (?)s^2 + (?)s + (?) = 0$		
Write this characteristic equation for the dominant poles model, (including the third pole at $s = -a$): $= 0$		
Write this characteristic equation for the actual closed loop transfer function, in terms of the PI Controller Gains K_p and K_i: $= 0$		
$a =$	$K_p =$	$K_i =$

Question 2 (Compulsory)

Controllability and Observability, System Eigenvalues, Transfer Function from State Space Model.

Consider a control system described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1-2\alpha & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & \alpha & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a) **(5 marks)** Is the system controllable? Does the Controllability depend on the value of parameter α? If so, find the condition for Controllability.

b) **(5 marks)** Is the system observable? Does the Observability depend on the value of parameter α? If so, find the condition for Observability.

c) **(5 marks)** Find the system eigenvalues.

d) **(5 marks)** Find the system transfer function, $G(s) = \dfrac{Y(s)}{U(s)}$

NOTE: Definitions for Controllability Matrix, $\mathbf{M_c}$, Observability Matrix, $\mathbf{M_o}$, and Transfer Function from State Space are given below.

$$\mathbf{M}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix} \qquad \mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} \qquad \mathrm{G(s)} = \mathbf{C} \cdot (\mathrm{s}\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$$

Question 2 Continued

Question 2 Continued

Question 2 Continued

Question 3

Closed Loop Stability, determining the range of safe operations under Proportional Control - Routh-Hurwitz and Bode Criteria.

Consider a certain control system under Proportional Control, as shown in Figure Q3.1. Open loop frequency response plots of the system, with Controller Gain K_p=1, are shown in Figure Q3.2.

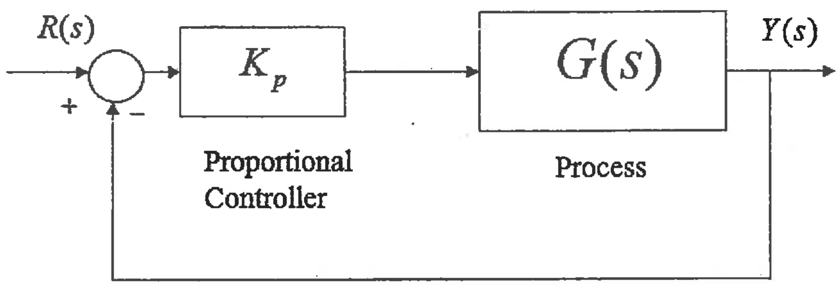

Figure Q3.1

1) **(6 marks)** Find the system Gain Margin, Phase Margin and corresponding crossover frequencies. Determine the critical value of the gain, K_{crit} , at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}. Determine the range of gains K_p to provide a stable closed loop system response and place your answers in Table Q3.1.

2) **(14 marks)** The process transfer function, G(s), is known, and given as follows:

$$G(s) = \frac{40}{s^3 + 11s^2 + 15s + 5}$$

Verify the results of Item 1) by applying the Routh-Hurwitz Criterion of Stability, i.e. find the critical value of the gain, K_{crit} , at which the system becomes marginally stable, and the corresponding frequency of marginally stable oscillations, ω_{osc}. How do they compare to the values in Item 1)? Place your answers in Table Q3.1.

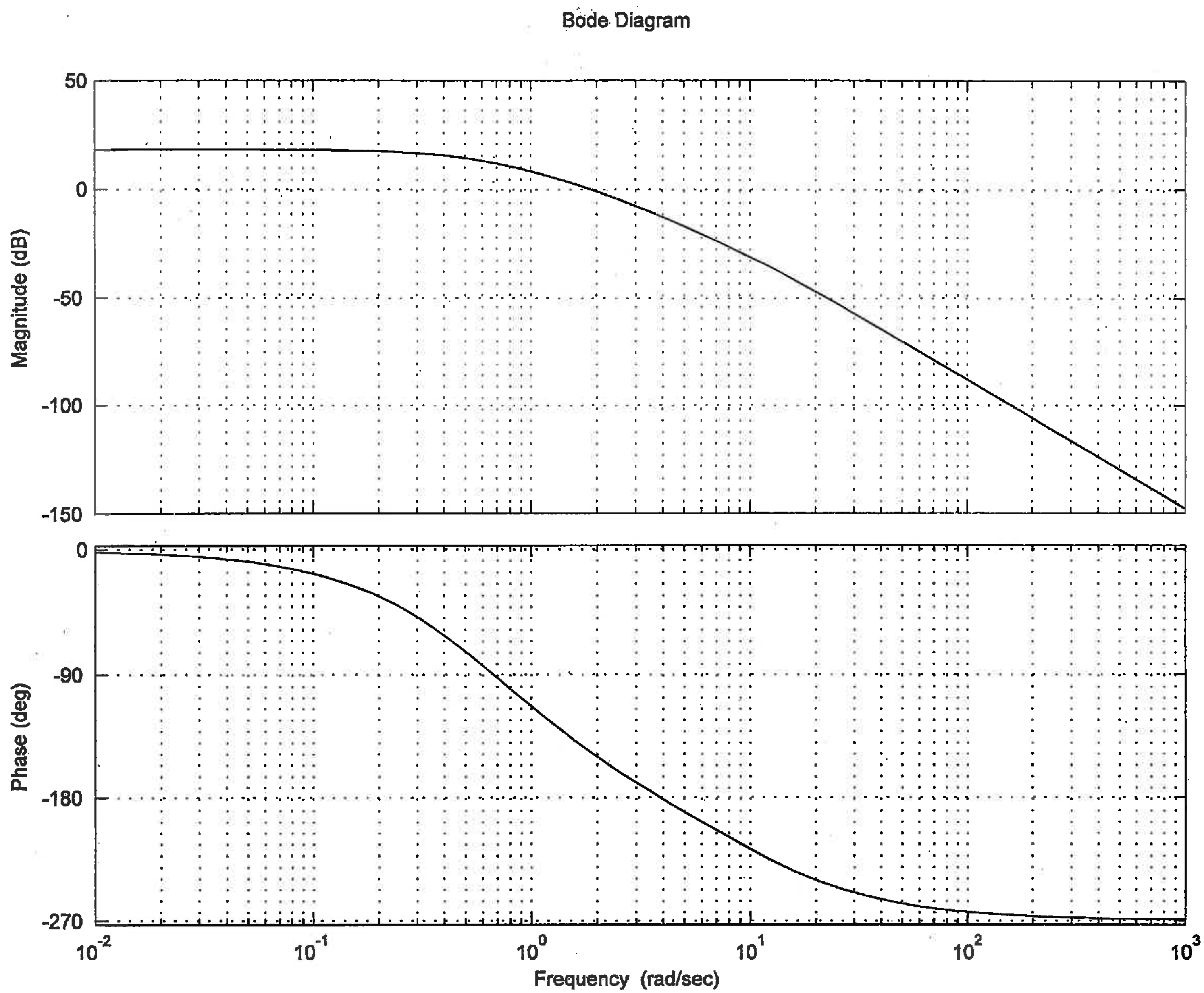
Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
0
-90
-180
-270
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

Figure Q3.2

Question 3 Continued

Use this page to do your analytical Routh-Hurwitz calculations.

Question 3 Continued

Table Q3.1

<table>
<tr><td colspan="2">Gain Margin</td><td colspan="2">Corresponding crossover frequency</td><td colspan="2">Phase Margin</td><td colspan="2">Corresponding crossover frequency</td><td colspan="2">Critical gain from Bode plot</td><td colspan="2">Frequency of oscillations from Bode plot</td></tr>
<tr><td colspan="2">$G_{m\,dB} =$</td><td colspan="2">$\omega_{cg} =$</td><td colspan="2">$\Phi_m =$</td><td colspan="2">$\omega_{cp} =$</td><td colspan="2">$K_{crit} =$</td><td colspan="2">$\omega_{osc} =$</td></tr>
<tr><td colspan="12">Range of gains for stable closed loop operation from Bode plot: $< K_p <$</td></tr>
<tr><td colspan="3">Critical gain From Routh-Hurwitz</td><td colspan="3">$K_{crit} =$</td><td colspan="3">Frequency of oscillations from Routh-Hurwitz</td><td colspan="3">$\omega_{osc} =$</td></tr>
<tr><td colspan="12">Range of gains for stable closed loop operation from Routh-Hurwitz Criterion: $< K_p <$</td></tr>
</table>

Question 4

Root Locus Analysis, Second Order Dominant Poles Model, Step Response Specifications, Proportional Controller Design.

Consider a certain closed loop control system under Proportional Control shown in Figure Q4.1:

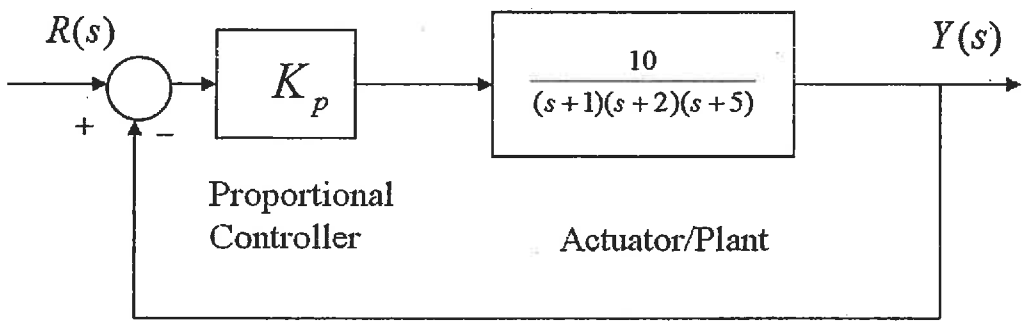

Figure Q4.1

1) **(8 marks)** In the space provided in Figure Q4.2, sketch a detailed Root Locus for the system, including crossovers with the imaginary axis, if any, break-away/break-in coordinates, if any, asymptotes, if any, a centroid, etc. Place the Root Locus parameters in Table Q4.1.

2) **(4 marks)** Assume that the closed loop system can be approximated by a second order model. Determine the appropriate value of the closed loop system equivalent damping ratio ζ so that the closed loop system step response displays a Percent Overshoot of approximately 15%. Next, use this information and the Root Locus sketch from Item 1) to determine the remaining second order model parameters K_{dc} and ω_n, and calculate the model transfer function $G_{model}(s)$. Place your answers in Table Q4.2.

3) **(5 marks)** Based on the Root Locus in Item 1) and on the results of Item 2), calculate the required value of the Proportional Gain K_p to achieve the desired closed loop system equivalent damping ratio ζ and find the actual closed loop transfer function at the calculated Gain, $G_{closed}(s)$. Place your answers in Table Q4.2.

3) **(3 marks)** Next, compare $G_{closed}(s)$ and $G_{model}(s)$. Does the model represent adequately the dominant dynamic of the actual closed loop system? Comment briefly on it in the lined space provided.

Table Q4.1

Root Locus centroid is at:	$\sigma =$
Root Locus asymptotic angles are equal to:	$\theta_i =$
The critical value of the Proportional Gain for which the system is marginally stable, and the frequency of resulting oscillations:	$K_{crit} =$ $\omega_{osc} =$
Break-away (leave blank if not applicable) coordinate is at:	$s_b =$
Break-in (leave blank if not applicable) coordinate is at:	$s_b =$

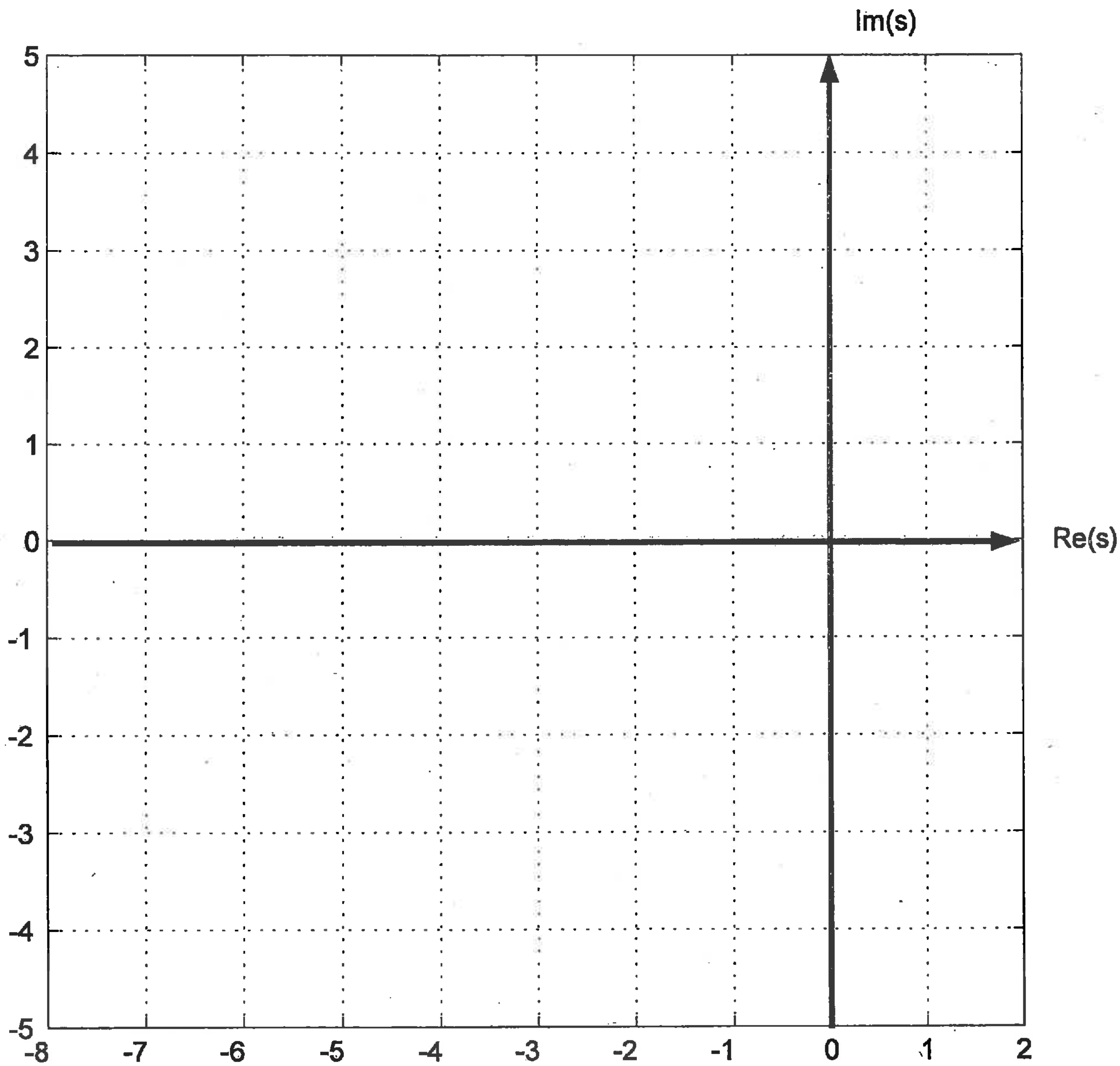

Figure Q4.2

Question 4 Continued

Question 4 Continued

Question 4 Continued

Table Q4.2

<table>
<tr><td>$\zeta =$</td><td>$\omega_n =$</td><td>$K_{dc} =$</td></tr>
<tr><td colspan="3">Assumed second order model:

$G_{model}(s) =$</td></tr>
<tr><td colspan="2">Controller gain required for $PO = 15\%$ is equal to:</td><td>$K_p =$</td></tr>
<tr><td colspan="3">Closed loop transfer function at this value of K_p:

$G_{closed}(s) =$</td></tr>
</table>

Question 5

Lead Controller Properties, Closed Loop Stability, Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications and Steady State Errors.

Consider the closed loop system shown in Figure Q5.1:

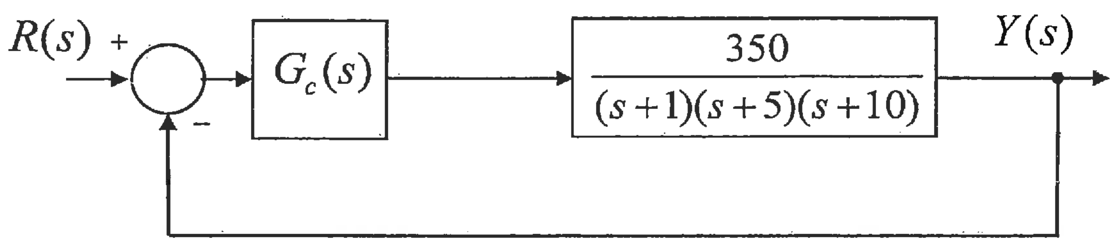

Figure Q5.1

Initially the system is not compensated; i.e. the controller transfer function is: $G_c(s) = 1$. Next, a LEAD Controller is designed for the system: $G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$. Frequency responses both for the uncompensated and the compensated system are shown in Figure Q5.2 - please note that based on Lead Controller characteristics, you should be able to recognize which trace corresponds to which response.

1) **(5 marks)** Apply the Bode criterion of stability to both the uncompensated and the compensated system, and place the appropriate values in Table Q5.1.

2) **(5 marks)** Based on the above, complete Table Q5.2 with estimated closed loop step response specifications for both the uncompensated and the compensated systems.

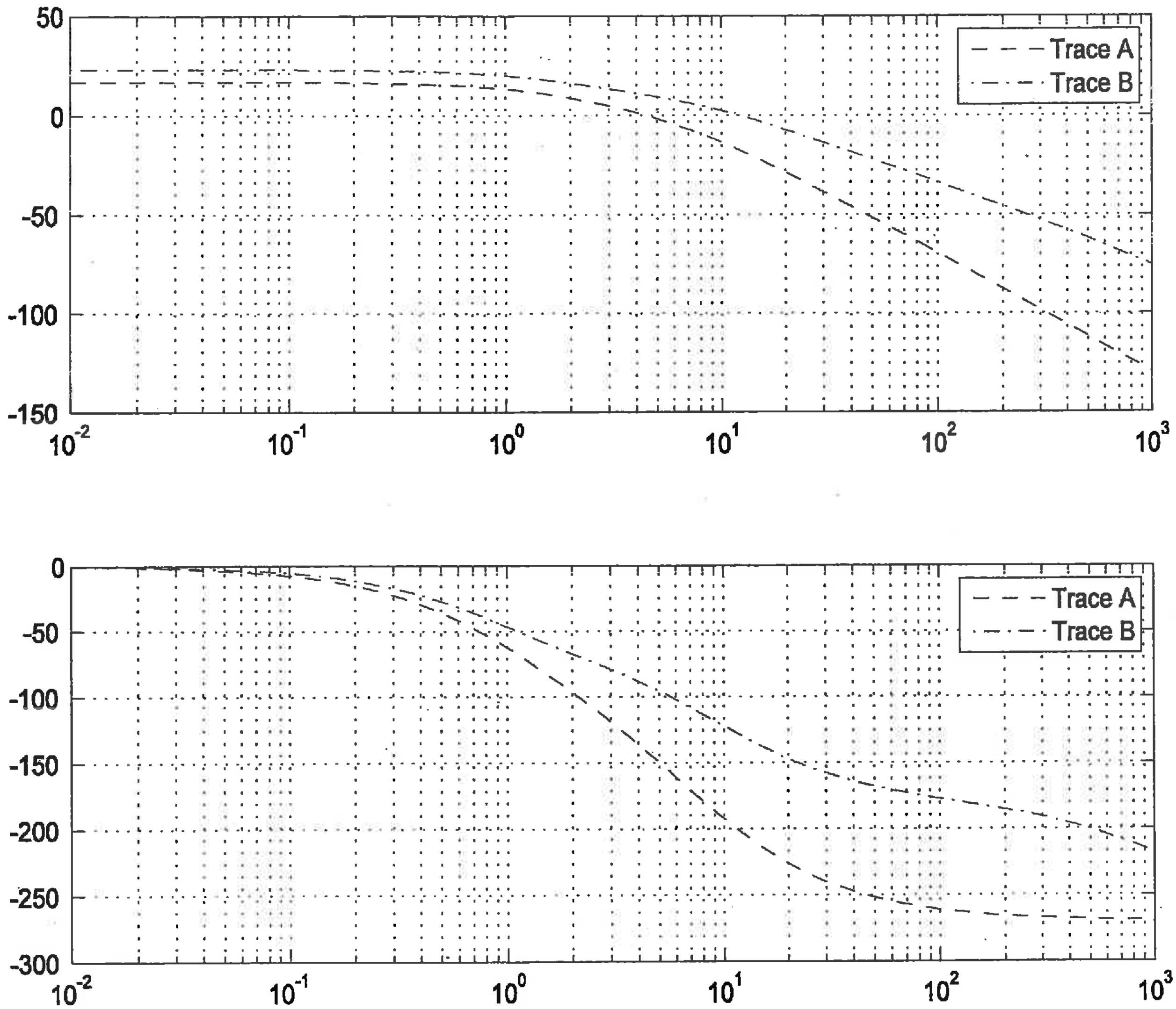

Figure Q5.1: Frequency Response
Note: Magnitude is in dB, Phase in Degrees, Frequency in rad/sec

Table Q5.1

	Gain Margin G_m in dB	Critical gain K_{crit} in V/V	Frequency of G_m crossover ω_{cg}	Phase Margin Φ_m in degrees	Frequency of Φ_m crossover ω_{cp}
Uncompensated system					
Compensated system					

Question 5 Continued

Question 5 Continued

Question 5 Continued

Table Q5.2

	Percent Overshoot, PO	Period of Oscillations, T_{period}	Settling Time $T_{settle\,(\pm 5\%)}$	Steady State Error $e_{ss(\%)}$
Uncompensated system				
Compensated system				

3) **(5 marks)** Based on Table Q5.2, has the relative stability of the system improved after the LEAD Controller was implemented, or worsened? Has the system step response improved after the LEAD Controller was implemented, or worsened? Place the checkmarks below:

	Improved	**Worsened**
Relative stability of Lead-compensated system:		
Transient response		
Steady state response		

4) **(5 marks)** Based on Table Q5.2, plot approximate shapes of the uncompensated and compensated step responses of the system in the space provided in Figure Q5.2.

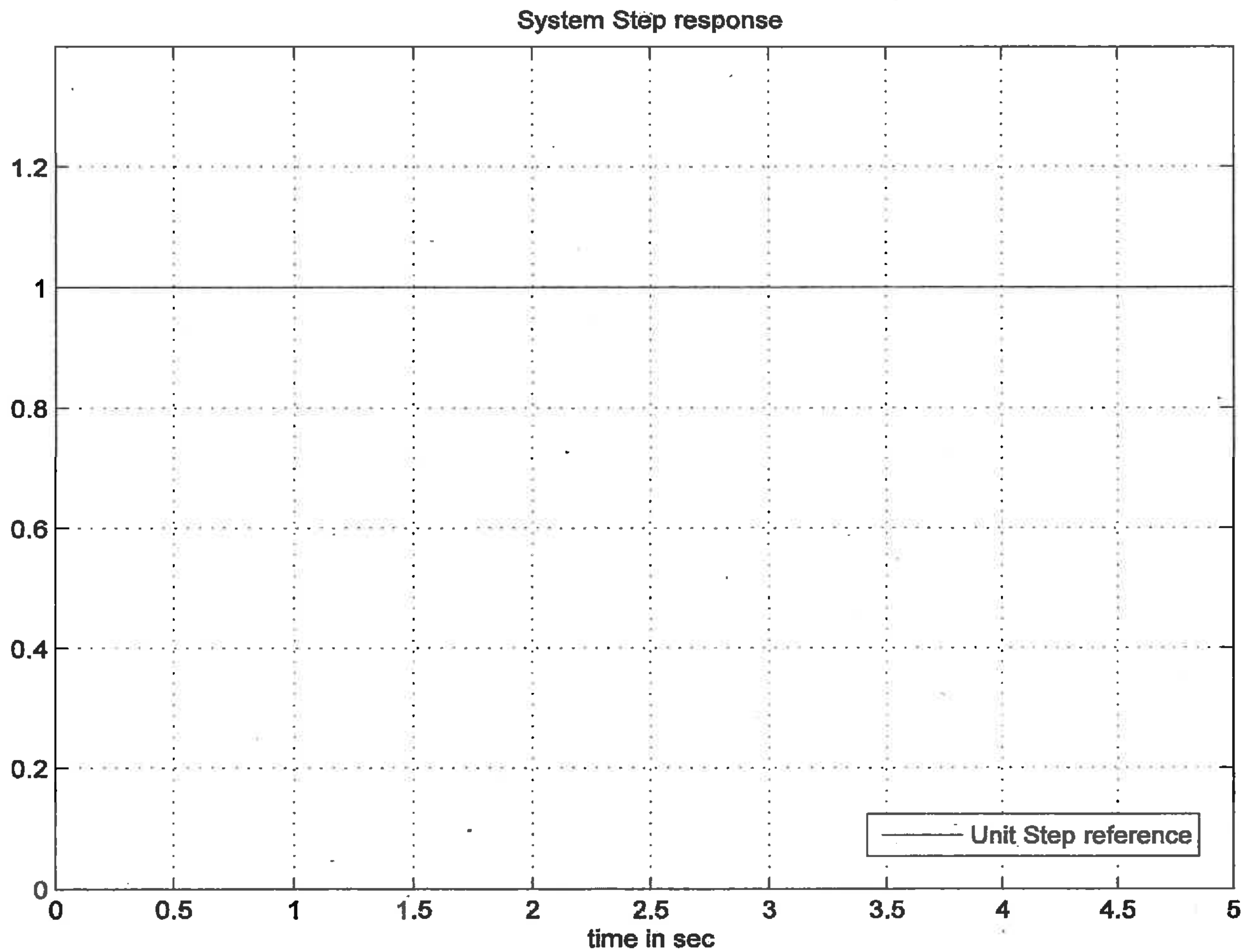

Figure Q5.2

Question 6

State Space Model from Transfer Functions, System Stability, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear process described by the signal flow graph in Figure Q6.1:

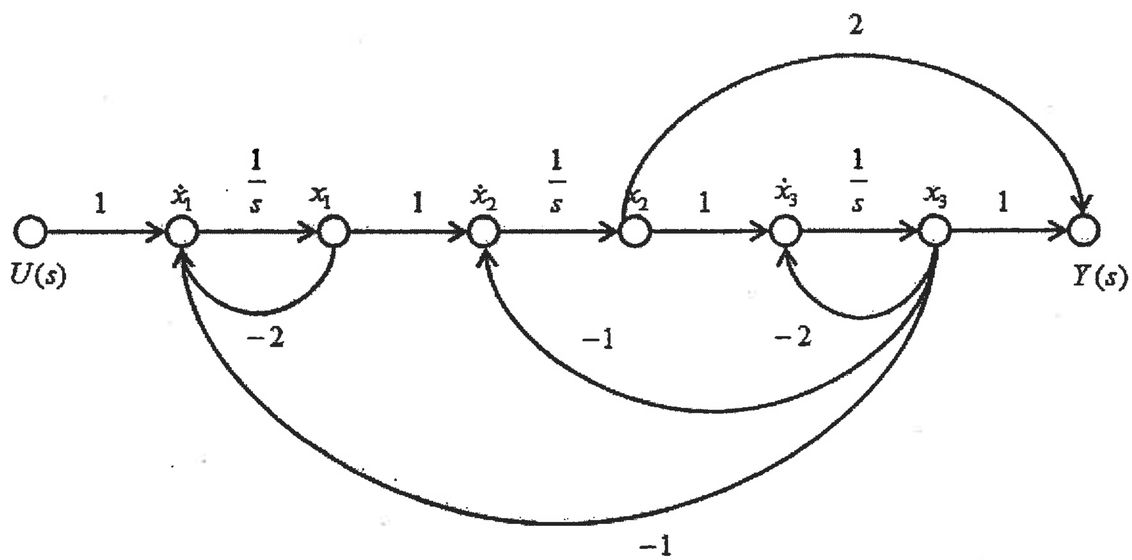

Figure Q6.1

1) **(6 marks)** Derive a set of the corresponding state equations - follow the choice of state variables as indicated in Figure Q6.1.

2) **(6 marks)** Determine if the process is stable.

3) **(8 marks)** A control system is to be built around the process by utilizing a state-variable feedback according to the following equation:

$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

Determine the values of the gain constant K and the state feedback vector $\mathbf{k}$ so that the closed loop system will have poles at: -12 and $-2 \pm 2j$, and the steady-state error to a step input is to be zero.

Question 6 Continued

Question 6 Continued

Question 6 Continued

Question 7

Steady State Errors, Error Constants and System Type. Second Order Model. Proportional Control.

Part A (10 marks)

Consider a response of a certain process $G(s)$ to a unit reference signal, shown in Figure Q7.1.

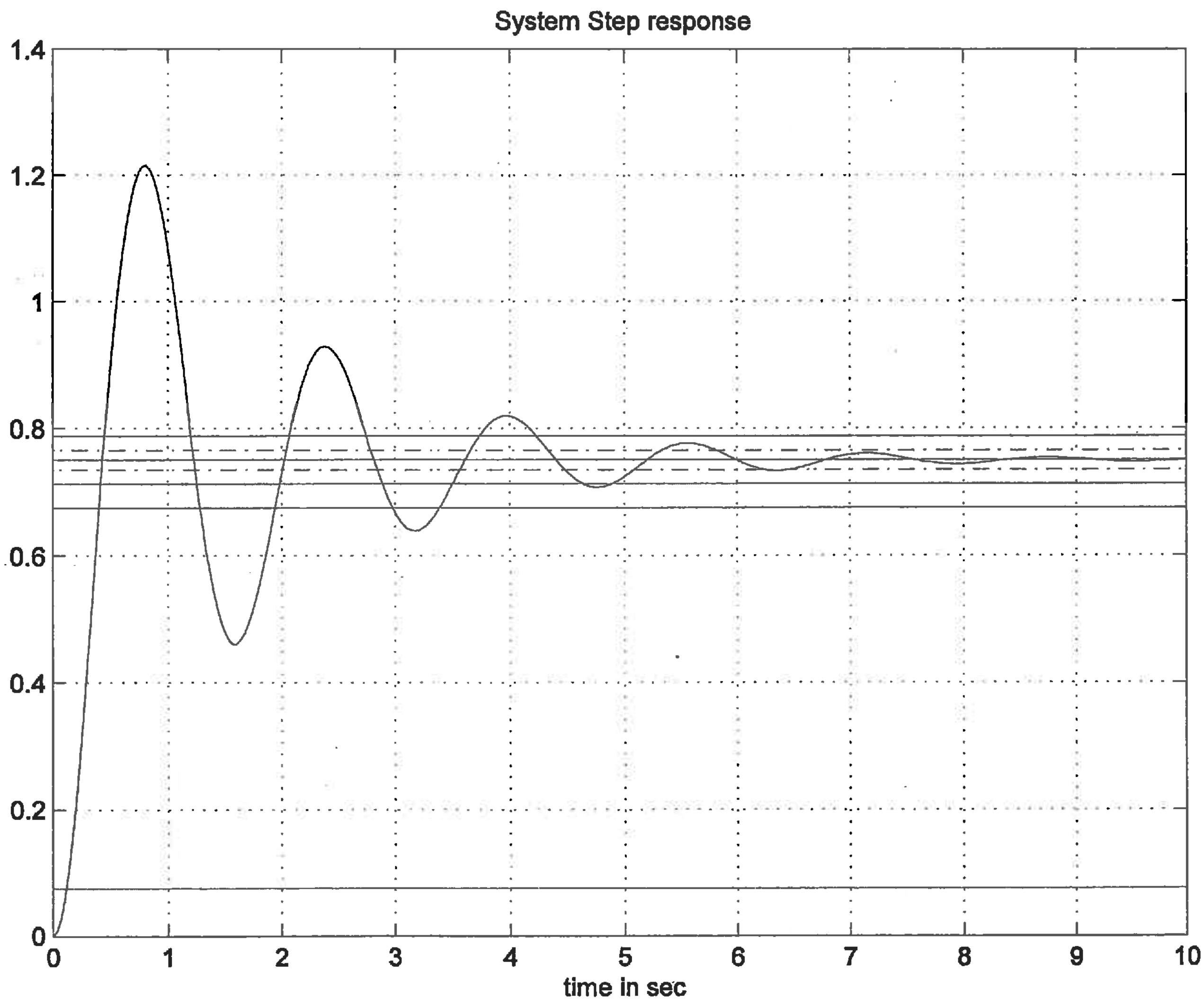

Figure Q7.1

1) **(5 marks)** Estimate the following specifications for the process $G(s)$ response to a unit input: Percent Overshoot, PO, Settling Time, $T_{settle(\pm 2\%)}$, and the steady state value of the process response, y_{ss}. Place your answers in Table Q7.1.

2) **(5 marks)** Based on the results of Item 1, assume that the process G(s) can be modeled by a standard second order system with a transfer function of the form as below:

$$G(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Find the process parameters, DC gain, K_{dc}, damping ratio, ζ, frequency of natural oscillations, ω_n, and write the process transfer function G(s). Place your answers in Table Q7.1.

Table Q7.1

The process G(s) step response specifications are as follows:		
$PO =$	$T_{settle\pm 2\%} =$	$y_{ss} =$
The process G(s) can be modeled as follows:		
$K_{dc} =$	$\zeta =$	$\omega_n =$
$G(s) = K_{dc}\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} =$		

Part B (10 marks)

1) **(6 marks)** Assume now that the process G(s) identified in Part A is part of a closed loop control system under Proportional Control, as shown in Figure Q7.2.

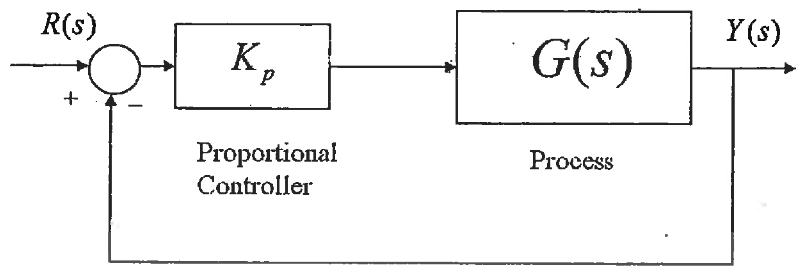

Figure Q7.2

What is the System Type of the closed loop transfer function $G_{cl}(s)$? Compute the system Error Constants and Steady State Errors (K_{pos}, $e_{ss(step\%)}$, K_v, $e_{ss(ramp)}$, K_a, $e_{ss(parab)}$) as functions of the Proportional Gain K_p. Next, compute the Proportional Controller Gain, K_p, such that the closed loop system Steady State Error for a step input, $e_{ss(step\%)}$, is equal to 5%. Place your answers in Table Q7.2.

2) **(4 marks)** What if we wanted to make the closed loop system Steady State Error for a step input to be arbitrarily small? What is the system Gain Margin? Is it going to limit how small the error can be made? Are there any other considerations? Discuss it briefly in the lined space provided.

Question 7 Continued

Table Q7.2

Closed Loop System Type: N =	$K_{pos} =$	$K_v =$	$K_a =$
	$e_{ss(step)\%} =$	$e_{ss(ramp)} =$	$e_{ss(parab)} =$
Proportional Controller Gain required for the 5% Steady State Error:		$K_p =$	