NATIONAL EXAMINATIONS MAY 2012

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks

2. (a) 15 marks ; (b) 5 marks

3. (a) 5 marks ; (b) 9 marks ; (c) 6 marks

4. 20 marks

5. 20 marks

6. (a) 10 marks ; (b) 10 marks

7. (a) 7 marks ; (b) 6 marks ; (c) 7 marks

1.Consider the following differential equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+(2-x^2)y=0$$

Find two linearly independent solutions about the ordinary point x=0.

2. (a) Find the Fourier series expansion of the periodic function f(x) of period p=2π.

$$f(x)=\begin{cases}1 & -\pi<x\leq 0\\ 2 & 0<x\leq\pi\end{cases}$$

(b) Use the result obtained in (a) to prove that $\frac{\pi}{4}=\sum_{n=1}^{\infty}\frac{(-1)^{(n+1)}}{2n-1}$

3.Consider the following function where a is a positive constant

$$f(x)=\begin{cases}\frac{1}{a}(1+\frac{x}{a}) & -a\leq x<0\\ \frac{1}{a}(1-\frac{x}{a}) & 0\leq x\leq a\end{cases}$$

(a) Compute the area bounded by f(x) and the x-axis. Graph f(x) against x for $a=0.2$ and $a=0.1$.

(b) Find the Fourier transform F(ω) of f(x).

(c) Graph F(ω) against ω for the same two values of a mentioned in (a). Explain what happens to f(x) and F(ω) when a tends to infinity.

Note: $$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)\exp(-i\omega x)dx$$

4. Set up Newton's divided difference formula for the data tabulated below and derive from it the polynomial of highest possible degree. Then compute F(−3) and F(5).

x	−6	−4	−1	0	1	3	6
F(x)	660	81	0	15	16	−24	126

5.The following results were obtained in a certain experiment:

x	−4.0	−3.0	−2.0	−1.0	0	1.0	2.0	3.0	4.0
f(x)	5.0	3.0	2.0	2.0	4.0	7.0	9.0	10.0	7.0

Use Romberg's algorithm to obtain an approximation of the area bounded by the unknown curve represented by the table and the lines x = −4, x = 4 and the x-axis.

Note: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(1,1)
R(2,1) R(2,2)
R(3,1) R(3,2) R(3,3)
R(4,1) R(4,2) R(4,3) R(4,4)

where

$$R(1,1) = \frac{H_1}{2}[f(a) + f(b)]$$

$$R(k,1) = \frac{1}{2}\left[R(k-1,1) + H_{k-1}\sum_{n=1}^{n=2^{k-2}} f(a+(2n-1)H_k)\right]; \qquad H_k = \frac{b-a}{2^{k-1}}$$

$$R(k,j) = R(k,j-1) + \frac{R(k,j-1) - R(k-1,j-1)}{4^{j-1}-1}$$

6. The equation $x^2 - 2 - 5\cos(x/2) = 0$ has a root between a=2 and b=3.

(a) Use the method of bisection four times to find a better approximation to this root.

(b) Starting with the last result obtained in (a) try to get a better approximation using the Newton-Raphson method twice. (Note: Carry seven digits in your calculations in the (b) part)

7. Consider the matrices $A = \begin{bmatrix} 2 & -2 & -2 \\ 4 & -4 & -2 \\ -2 & 1 & -1 \end{bmatrix}$; $U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) Prove that the matrix A satisfies the following equation

$$A^3 + 3A^2 - 4U = O \qquad (1)$$

(b) Equation (1) can be rewritten as follows

$$U = \frac{1}{4}(A^3 + 3A^2) \qquad (2)$$

Pre-multiplying both sides of equation (2) by A^{-1} we get

$$A^{-1} = \frac{1}{4}(A^2 + 3A) \qquad (3)$$

Use equation (3) to find A^{-1}.

(c) Use the result obtained in (b) to solve the following system of three linear equations:

$$\begin{aligned} 2x_1 - 2x_2 - 2x_3 &= 3 \\ 4x_1 - 4x_2 - 2x_3 &= 12 \\ -2x_1 + x_2 - x_3 &= -10 \end{aligned}$$