

National Exams May 2012

**04-BS-6: Mechanics of Materials**

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

- ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae.  
**Example problems and solutions to problems are not allowed!**
3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
  4. All questions are of equal value.

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**NOTE: The aid sheet must be handed in with the exam!**

*Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.*

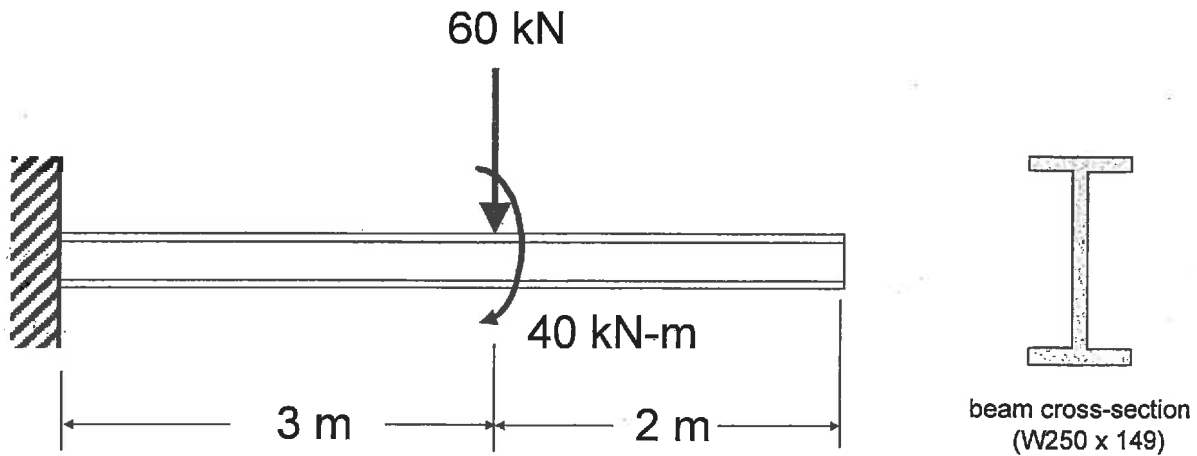
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**Question 1:** A cantilevered beam (fixed at the left) supports a concentrated load of 60 kN and [20 marks] a 40 kN-m couple both acting 3 m from the fixed support of the beam as shown. The beam is a W250 x 149 wide-flange section and is made from steel having an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel used equals 200 GPa. Refer to the attached table for section properties.

(a) determine the maximum deflection of the beam using the method of integration

(b) determine the slope at the free end of the beam

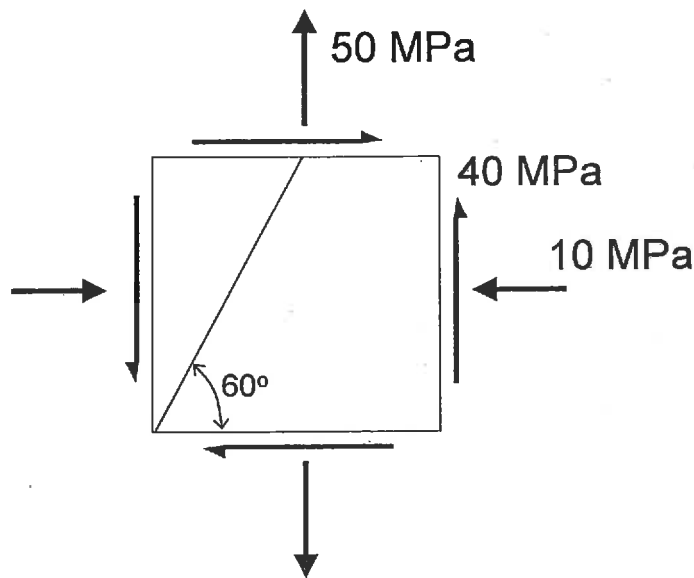
[bonus] (c) indicate whether the maximum deflection computed is reasonable or not and give reasons for your answer



No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

**Question 2:** For an element in a state of plane stress subjected to the normal and shear stresses [20 marks] shown below, use the Mohr's circle solution (*not* the transformation equations) to determine the following:

- (a) the stress components acting on the inclined plane (orientated  $60^\circ$  from the horizontal as shown), showing your answer on a properly oriented element.
- (b) the maximum in-plane shear stress of the element (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.

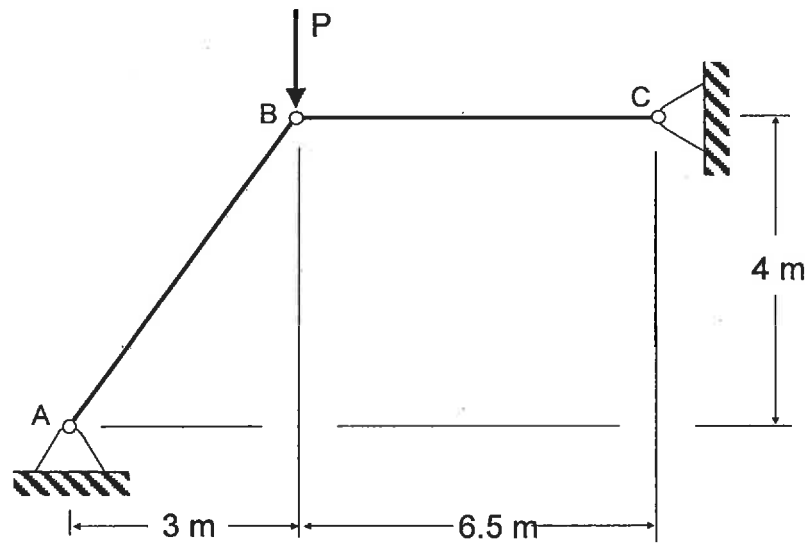


Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

**Question 3:** The truss ABC is constructed from two steel rods and supports a vertical load P applied at B as shown below. The two rods (AB and BC) have a 120 mm diameter and are pinned at both ends. The rods are made of steel with a yield strength of 350 MPa and elastic modulus of 200 GPa.

[20 marks]

Determine the largest load the structure can support. Consider in-plane buckling only and use a safety factor of 2 for Euler buckling. Do not use a safety factor for yielding of the steel.

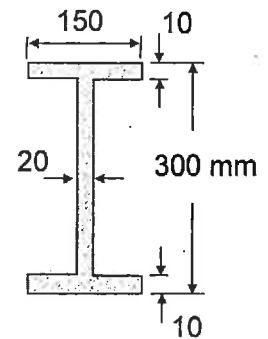
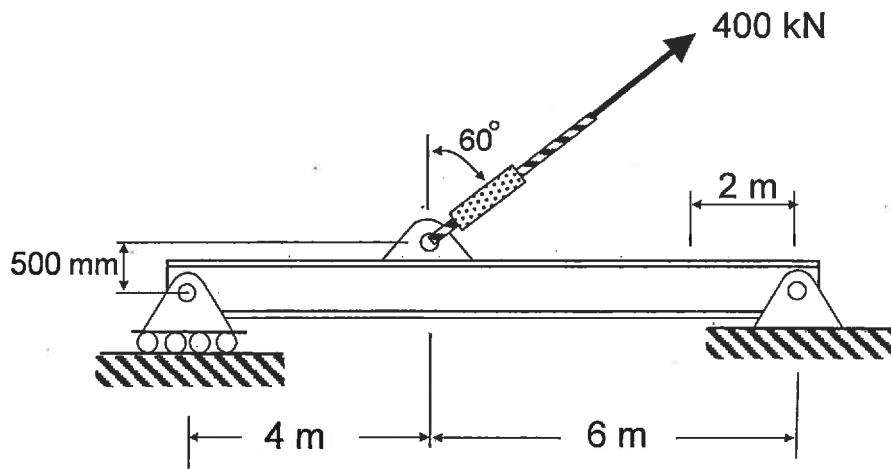


Note:  $A_{\text{circle}} = \pi r^2$  and  $I_{\text{circle}} = \pi r^4 / 4$

**Question 4:** A simply supported beam with the cross section shown is pulled with a cable attached to a connection plate located 4 m from the left support. The beam is made of steel with a yield stress of 310 MPa and elastic modulus of 200 GPa.

[20 marks]

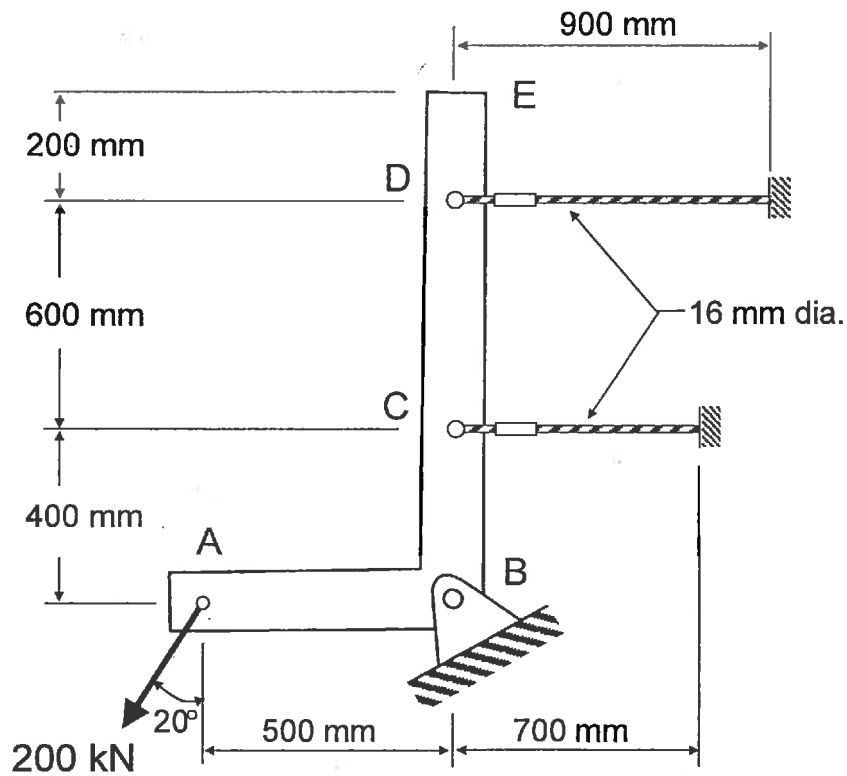
- (a) compute the normal stress distribution at a section located 2 m from the right support. Show your answer on a sketch and make sure to show maximum and minimum values of stress.
- (b) compute the maximum shear stress at a section located 2 m from the right support.



beam cross-section  
(all dimensions in mm)

**Question 5:** The rigid L shaped link (ABCDE) is supported by a pin at B and two cables located at C and D. The cables have a 16 mm diameter and are made of high strength steel with a yield strength of 800 MPa and elastic modulus of 200 GPa. The rigid link is designed to resist an inclined load equal to 200 kN (acting 20 degrees from the vertical as shown).

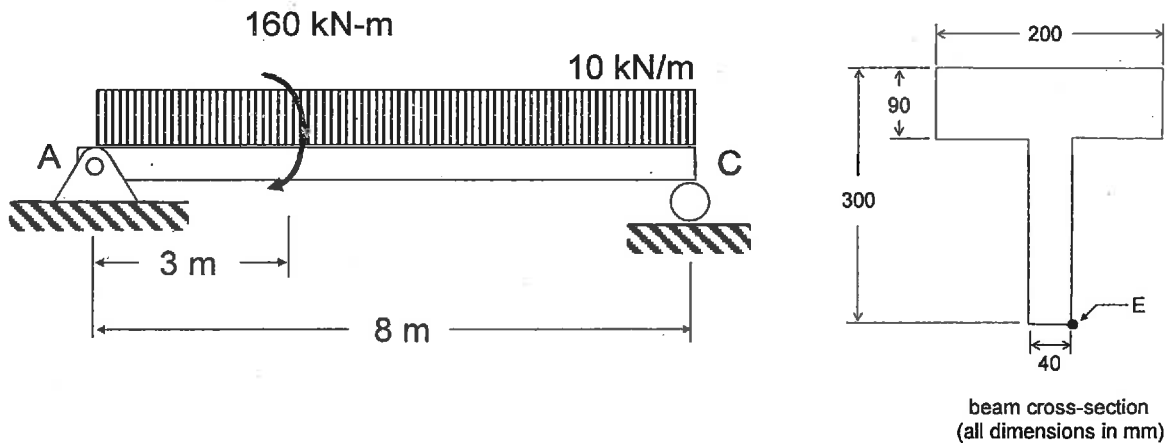
- [12 marks] (a) find the forces developed in each cable
- [4 marks] (b) find the corresponding vertical displacement at point A where the load is applied
- [4marks] (c) find the shear stress in the pin at B given that the pin has a diameter of 60 mm and is loaded in double shear



**Question 6:** A simply supported beam is subjected to a uniformly distributed load over the member span in addition to a concentrated moment acting 3 m from the left hand support as shown. The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.  
 [20 marks]

Determine the shear and moment throughout the beam as functions of  $x$ . This means that you need to give formula(s) for  $V(x)$  and  $M(x)$  along the length of the beam.

Next construct the shear force and bending moment diagrams. Remember to label points of maximum positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

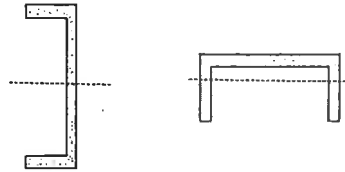


Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

**No credit** will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

**Question 7:** For the simply supported T beam and loading conditions from Question 6, do the [20 marks] following:

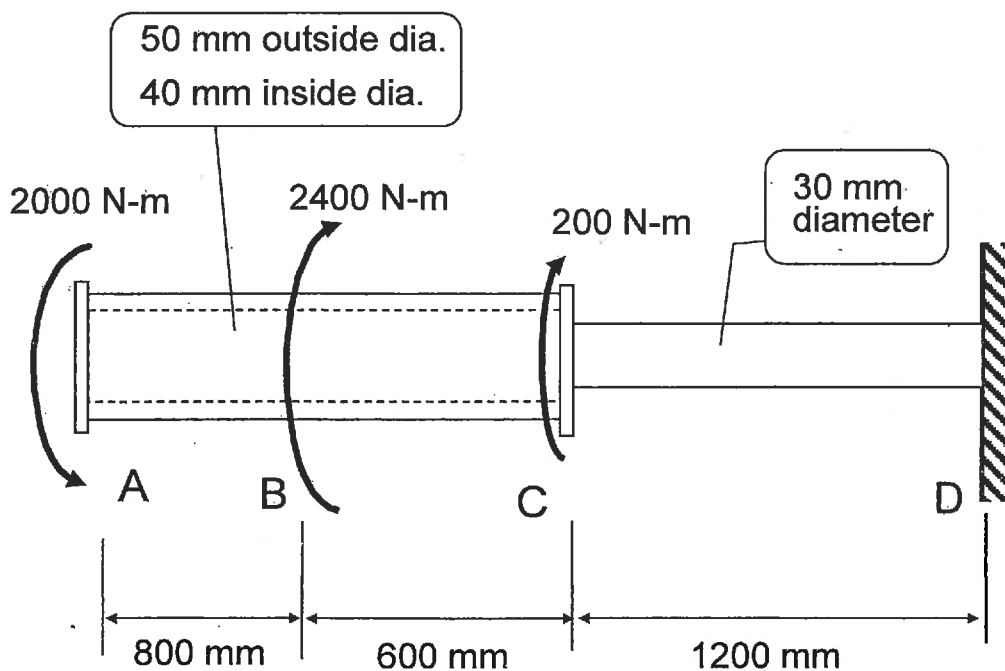
- (a) determine the maximum (absolute) normal stress and maximum shear stress in the beam.
- (b) determine the shear stress at the bottom of the web (point E). To receive marks you must give reasons to justify your answer.
- (c) for the two cross-sections shown below, indicate whether either section will experience any twisting (or torque) when bending about the horizontal axis. Give reasons for your answer.

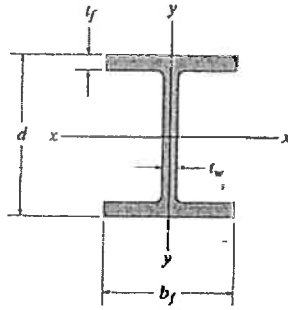




**Question 8:** A circular stepped shaft is subjected to three torques acting as shown below. Part of the shaft is hollow and the entire shaft is made of steel with  $G = 80 \text{ GPa}$  and a yield stress of  $150 \text{ MPa}$ . Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

- [12 marks] (a) determine the maximum shear stress in the stepped shaft and sketch the corresponding variation of shear stress along the shaft radius at this point.
- [6 marks] (b) find the angle of twist at the end of the shaft (point A) and give your answer in degrees.
- [2 marks] (c) A shoulder fillet is used at the step in the shaft (point C) to reduce the stress concentration factor  $K$  to a value of 1.4. Is this sufficient to avoid failure in the shaft?





Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t <sub>w</sub>	Flange		x-x axis			y-y axis		
				width b <sub>f</sub>	thickness t <sub>f</sub>	I	S	r	I	S	r
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0