

National Exams May 2012

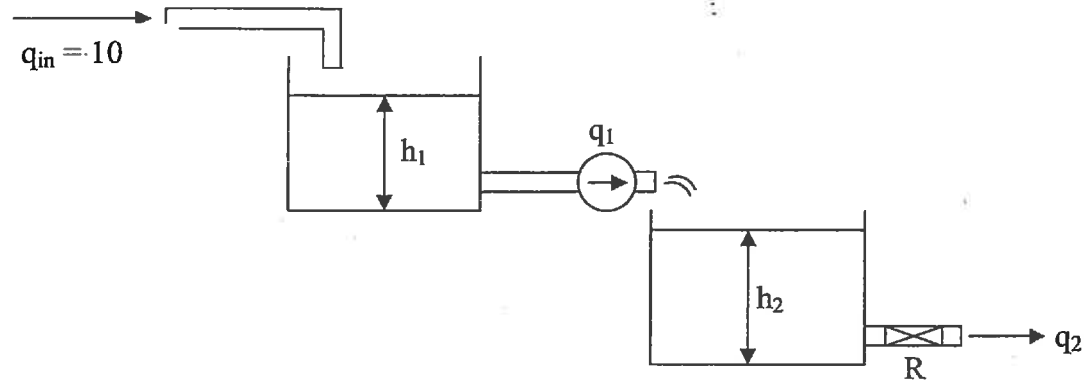
**04-Chem-A6, Process Dynamics and Control**

3 hours duration

**NOTES:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK EXAM.  
Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.  
The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Most questions require an answer in essay format. Clarity and organization of the answer are important.

**Problem #1** (20% total)



Two tanks are connected in series in a noninteracting fashion as shown in the figure.

Assume:  $\rho = 1$   $A = 1$  ( $A$ -cross-section of each tank)

$$q_2 = \frac{1}{R} \sqrt{\frac{\Delta P}{\rho g}} \text{ and } q_1 \text{ is determined by a pump.}$$

$q_{in} = 10$  and remains constant. The initial level in tank 1 is  $h_1(t=0) = 10$ .  $q_1$  is the manipulated variable. All  $q$ 's are volumetric flow rates.  $R = 2$ .

- (10%) (a) Show the differential equations that describe the behaviour of  $h_1(t)$  and  $h_2(t)$ .
- (10%) (b) Compute transfer functions between  $h_1$  to  $q_{in}$  and  $h_2$  to  $q_{in}$ .

**Problem #2** (20% total)

A process is described by the following transfer function:

$$G_p = \frac{10(0.5 - s)e^{-10s}}{100s + 1}$$

- (10%) (a) Design an IMC controller for this process. Show your design using a block diagram. Select the IMC filter to be of 1<sup>st</sup> order with a time constant  $\tau = 5$ .
- (10%) (b) Compute and plot qualitatively the closed loop response of the system to a unit step in set point. Assume a perfect model (i.e. no model error).

**Problem #3:** (20% total)

A second order process is given by

$$G_p(s) = \frac{1}{s^2 + 3s + 2}$$

This process is controlled by a proportional-derivative (PD) controller given by:

$$G_c(s) = k_c(1 + s)$$

- (10%) (a) Compute values of the  $k_c$  that will result in closed loop stability. Use Routh-test.
- (10%) (b) If instead of a PD, a proportional only controller  $G_c(s) = 0.1$  is used, compute the closed loop response as a function of time for a unit step change in set point.

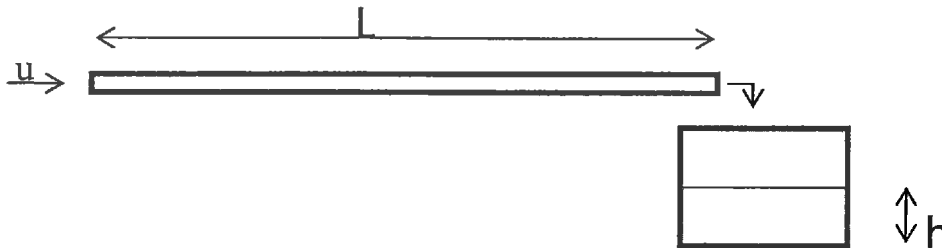
**Problem 4 (20% of total)**

For a second order process:

$$G(s) = \frac{1}{\tau^2 s^2 + 2\tau\xi s + 1} \quad \text{assume } \xi \text{ and } \tau \text{ are both positive}$$

- (5%) (a)- Find the frequency response  $B(\omega)$  and  $\Phi(\omega)$  for an input  $A \cdot \sin(\omega t)$  for  $\xi < 1$ .
- (5%) (b)- Plot qualitatively the Bode plots for the amplitude ratio and the phase angle for  $\xi < 1$ . Show asymptotes for very small and very large values of  $\omega\tau$ , show value of slope of the asymptote in logarithmic scale, show "corner" frequency.
- (10%) (c)- If a proportional controller with gain  $K_c$  is used to control this process, what is the Largest  $K_c$  that can be used for stability? (use a stability test of your choice)

**Problem 5 (20% of total)**



A pipe of length  $L=10$  m is feeding liquid into a tank. The velocity of the liquid in the pipe is  $u=1$  m/s. Assume plug flow. The level of liquid in the tank is  $h$ . The cross section area of the tank is  $A=1$  m<sup>2</sup>.

- (5%) (a)- Model the system, i.e. formulate differential equations to calculate  $h(t)$  with respect to inlet volumetric flowrate to the pipe  $q$  and find the open loop transfer function between  $h$  to  $q$ .
- (5%) (b)- If the height  $h$  is controlled by manipulating the flowrate  $q$  with a proportional controller with gain  $K_c$ , find the closed loop transfer function between the set point changes in  $h$  to changes in  $h$ .
- (10%) (c)- What is the largest gain  $K_c$  for stability. Do not use Pade approximation. Then, calculate the gain margin for  $K_c=0.1$ .

**Problem #6** (20% total)

The dynamic response of the reactant concentration  $C_A$  in a CSTR reactor, to a change in inlet concentration  $C_{A_i}$ , has to be evaluated. The reactor is operated with constant volume  $V$  and at isothermal conditions. The density of the liquid inside the reactor  $\rho$  is constant.

The reaction rate is:

$$r_A = k_1 C_A^2$$

The mass flow rate is  $F$ .

- (10%) (a) Derive a mathematical model for  $C_A$  as a function of time.
- (10%) (b) Compute an approximate transfer function  $\delta C_A / \delta C_{A_i}$ , where  $\delta$  indicates deviation variables, i.e. changes in exit concentration to inlet concentration, when the system is operated close to a steady state corresponding to an inlet concentration of  $C_{A_i}^0$ . Express the transfer function as a function of  $\rho$ ,  $V$ ,  $C_{A_i}^0$ ,  $F$  and  $k_1$ .

**PROBLEM # 7** (20% of total)

A process given by:

$$G_p = \frac{100}{s-10}$$

is controlled by a proportional controller with gain  $k_c$ .

- (10%) (a) Using the Nyquist theorem test the closed loop stability for  $k_c = 1$  and  $k_c = 0.01$ .
- (10%) (b) Using the Nyquist criterion, compute the limiting value of  $k_c$  for which the system is stable.

**Problem #8** (20% total)

For the process modelled by:

$$\frac{dy_1}{dt} = -y_1 + y_2 + x_1$$

$$\frac{dy_2}{dt} = y_1 - 2y_2 + x_1 + x_2$$

- (10%) (a) Find the two transfer functions relating the inputs  $x_1$  and  $x_2$  to the output  $y_2$ . The  $x$ 's and  $y$ 's are deviation variables.
- (10%) (b) Compute  $y_2$  as a function of time for  $x_1 = 0$  and a unit step in  $x_2$ .