National Exams
07-Elec-B1, Digital Signal Processing

May 2012

3 Hours Duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM, a Casio or Sharp approved calculator is permitted.
3. FOUR(4) questions constitute a complete paper. The first four questions as they appear in the answer book will be marked.
4. All questions are of equal value.
5. Clarity and organization of the answer are important.
6. LIST OF IMPORTANT EQUATIONS CAN BE FOUND ON LAST PAGE

1. **(25 marks total)** For the following input signals $x[n]$, find the system response, $y[n]$ (zero-state) of a Linear-time Invariant Discrete (LTID) system if its impulse response $h[n] = (0.5)^n u[n]$.

   (a) **(9 marks)** $x[n] = 2^n u[n]$

   (b) **(8 marks)** $x[n] = 2^{n-2} u[n]$

   (c) **(8 marks)** $x[n] = 2^n u[n-3]$

2. **(25 marks total)** For a Discrete system specified by the equation

$$y[n+1] - 0.8y[n] = x[n+1] \tag{1}$$

   Find the system response to the input

   (a) **(10 marks)** 1^n

   (b) **(8 marks)** $\cos[\frac{\pi}{6}n - 0.2]$

   (c) **(7 marks)** A sampled sinusoid $\cos 1500t$ with sampling interval $T = 0.001$.

3. **(25 marks total)** A causal linear time invariant system has the system function

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})} \tag{2}$$

   (a) **(6 marks)** Write the difference equation that is satisfied by the input and the output of the system.

   (b) **(6 marks)** Plot the pole-zero diagram and indicate the region of convergence for the system function.

   (c) **(6 marks)** Sketch $|H(e^{j\omega})|$.

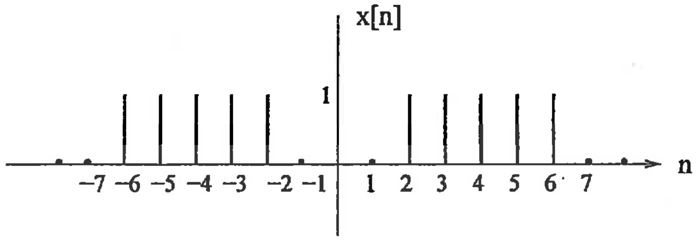
   (d) **(7 marks)** Determine the impulse response of the overall system in closed form.

4. **(25 marks total)**

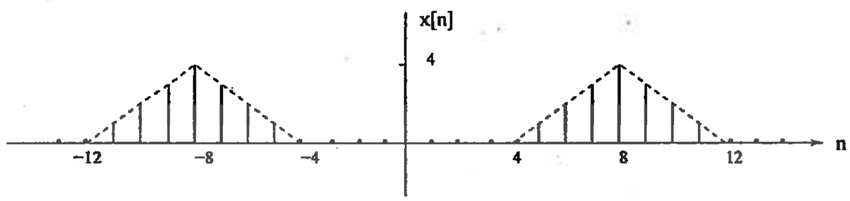
(5 marks) Use the time-shift property to show that

$$x[n+k] + x[n-k] \Leftrightarrow 2X(\Omega)\cos k\Omega \qquad (3)$$

(10 marks each) Use this result to find the DTFT of the signals shown in the following figures.



(a)



(b)

5. **(25 marks total)** The impulse response of a linear time-invariant system is

$$h[n] = \begin{cases} a^n, & 0 \leq n \leq 7 \\ 0, & otherwise \end{cases} \tag{4}$$

(a) **(6 marks)** Draw the flow graph of direct-form nonrecursive implementation of the system.

(b) **(6 marks)** Show that the corresponding system function can be expressed as

$$H(z) = \frac{1 - a^8 z^{-8}}{1 - az^{-1}}, \qquad |z| > |a|.$$

(c) **(5 marks)** Draw the flow graph of an implementation of $H(z)$, as expressed in part (b), corresponding to a cascade of an FIR system (numerator) with an IIR system (denominator).

(d) **(4 marks)** Is the implementation in part (c) recursive or nonrecursive? Is the overall system FIR or IIR?

(e) **(4 marks)** Which implementation of the system requires

i. The most storage (delay elements)?

ii. The most arithmetic (multiplications and additions per output sample)?

Trigonometric Identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin\alpha\,\sin\beta = \tfrac{1}{2}\cos(\alpha-\beta) - \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\cos\alpha\,\cos\beta = \tfrac{1}{2}\cos(\alpha+\beta) + \tfrac{1}{2}\cos(\alpha-\beta)$$
$$\sin\alpha\,\cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta)$$

Euler's Identity:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Discrete Fourier Transform:

The N-point DFT of a N-sample sequence $s[n]$ is defined as:

$$S[k] = \sum_{n=0}^{N-1} s[n]\,W_N^{kn}, \qquad k = 0, 1, \ldots, N-1.$$

The sequence $s[n]$ can be recovered from its DFT coefficients using the N-point IDFT:

$$s[n] = \frac{1}{N}\sum_{k=0}^{N-1} S[k]\,W_N^{-kn}, \qquad n = 0, 1, \ldots, N-1.$$

where $W_N = e^{-j2\pi/N}$. The DFT/IDFT relations can also be expressed in matrix form

$$\begin{bmatrix} S[0] \\ \vdots \\ S[N-1] \end{bmatrix} = \mathbf{W}_N \begin{bmatrix} s[0] \\ \vdots \\ s[N-1] \end{bmatrix} \qquad \begin{bmatrix} s[0] \\ \vdots \\ s[N-1] \end{bmatrix} = \mathbf{W}_N^{-1} \begin{bmatrix} S[0] \\ \vdots \\ S[N-1] \end{bmatrix}$$

where the transformation matrices for $N = 2, 3, 4$ are

$$\mathbf{W}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{W}_2^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{W}_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - \frac{\sqrt{3}}{2}j & -\frac{1}{2} + \frac{\sqrt{3}}{2}j \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}}{2}j & -\frac{1}{2} - \frac{\sqrt{3}}{2}j \end{bmatrix} \qquad \mathbf{W}_3^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}}{2}j & -\frac{1}{2} - \frac{\sqrt{3}}{2}j \\ 1 & -\frac{1}{2} - \frac{\sqrt{3}}{2}j & -\frac{1}{2} + \frac{\sqrt{3}}{2}j \end{bmatrix}$$

$$\mathbf{W}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \qquad \mathbf{W}_4^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}$$

TABLE 9.1 A Short Table of Discrete-Time Fourier Transforms

No.	$x[n]$	$X(\Omega)$	
1	$\delta[n-k]$	$e^{-jk\Omega}$	Integer k
2	$\gamma^n u[n]$	$\dfrac{e^{j\Omega}}{e^{j\Omega}-\gamma}$	$\lvert\gamma\rvert < 1$
3	$-\gamma^n u[-(n+1)]$	$\dfrac{e^{j\Omega}}{e^{j\Omega}-\gamma}$	$\lvert\gamma\rvert > 1$
4	$\gamma^{\lvert n\rvert}$	$\dfrac{1-\gamma^2}{1-2\gamma\cos\Omega+\gamma^2}$	$\lvert\gamma\rvert < 1$
5	$n\gamma^n u[n]$	$\dfrac{\gamma e^{j\Omega}}{(e^{j\Omega}-\gamma)^2}$	$\lvert\gamma\rvert < 1$
6	$\gamma^n\cos(\Omega_0 n+\theta)u[n]$	$\dfrac{e^{j\Omega}[e^{j\Omega}\cos\theta-\gamma\cos(\Omega_0-\theta)]}{e^{j2\Omega}-(2\gamma\cos\Omega_0)e^{j\Omega}+\gamma^2}$	$\lvert\gamma\rvert < 1$
7	$u[n]-u[n-M]$	$\dfrac{\sin(M\Omega/2)}{\sin(\Omega/2)}e^{-j\Omega(M-1)/2}$	
8	$\dfrac{\Omega_c}{\pi}\operatorname{sinc}(\Omega_c n)$	$\sum_{k=-\infty}^{\infty}\operatorname{rect}\left(\dfrac{\Omega-2\pi k}{2\Omega_c}\right)$	$\Omega_c \le \pi$
9	$\dfrac{\Omega_c}{2\pi}\operatorname{sinc}^2\left(\dfrac{\Omega_c n}{2}\right)$	$\sum_{k=-\infty}^{\infty}\Delta\left(\dfrac{\Omega-2\pi k}{2\Omega_c}\right)$	$\Omega_c \le \pi$
10	$u[n]$	$\dfrac{e^{j\Omega}}{e^{j\Omega}-1}+\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$	
11	1 for all n	$2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$	
12	$e^{j\Omega_0 n}$	$2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)$	
13	$\cos\Omega_0 n$	$\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)+\delta(\Omega+\Omega_0-2\pi k)$	
14	$\sin\Omega_0 n$	$j\pi\sum_{k=-\infty}^{\infty}\delta(\Omega+\Omega_0-2\pi k)-\delta(\Omega-\Omega_0-2\pi k)$	
15	$(\cos\Omega_0 n)u[n]$	$\dfrac{e^{j2\Omega}-e^{j\Omega}\cos\Omega_0}{e^{j2\Omega}-2e^{j\Omega}\cos\Omega_0+1}+\dfrac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k-\Omega_0)+\delta(\Omega-2\pi k+\Omega_0)$	

TABLE 9.2 Properties of the DTFT

Operation	$x[n]$	$X(\Omega)$
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(\Omega)+a_2X_2(\Omega)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Scalar multiplication	$ax[n]$	$aX(\Omega)$
Multiplication by n	$nx[n]$	$j\dfrac{dX(\Omega)}{d\Omega}$
Time reversal	$x[-n]$	$X(-\Omega)$
Time shifting	$x[n-k]$	$X(\Omega)e^{-jk\Omega}$ k integer
Frequency shifting	$x[n]e^{j\Omega_c n}$	$X(\Omega-\Omega_c)$
Time convolution	$x_1[n]*x_2[n]$	$X_1(\Omega)X_2(\Omega)$
Frequency convolution	$x_1[n]x_2[n]$	$\dfrac{1}{2\pi}\int_{2\pi}X_1[u]X_2[\Omega-u]\,du$
Parseval's theorem	$E_x=\sum_{-\infty}^{\infty}\lvert x[n]\rvert^2$	$E_x=\dfrac{1}{2\pi}\int_{2\pi}\lvert X(\Omega)\rvert^2\,d\Omega$