NATIONAL EXAMS May 2012
07-Elec-B2 Advanced Control Systems
3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio or Sharp approved model.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

1. The state space model for a linear system is parameterized by,

$$A=\begin{bmatrix}0 & 6\\ -6 & -12\end{bmatrix},\ B=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix},\ C=\begin{bmatrix}3 & 0\end{bmatrix},\ D=\begin{bmatrix}0 & 0\end{bmatrix}$$

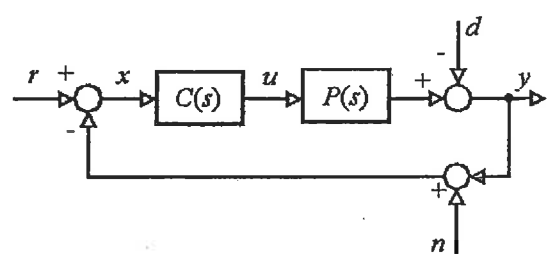
(a) Determine the system poles.
(b) Find the transfer function that relates the system output to the input(s).
(c) Find an expression for the output, $y(t)$, when the input is zero and the initial condition is $x(0)=\begin{bmatrix}0 & 1\end{bmatrix}^T$.

2. Consider the system, $y(s)=G(s)u(s)$, $G(s)=\dfrac{1-\alpha s}{(2+s)(1+s)}$.

(a) Find a state space model for the system taking $y(t)$ as one of the state variables.
(b) Determine the conditions under which the system is controllable and observable.
(c) With $\alpha=1$, design a state feedback controller such that the closed loop poles are -10, -5.

3. Consider the control system below with, $P(s)=\dfrac{10}{(s+1)^2}$, $C(s)=\dfrac{sK_p+K_i}{s}$

(a) Design a proportional-integral control system such that the phase margin is at least 45 degrees and the gain crossover is as large as feasible.
(b) Sketch the magnitude of the Bode plot associated with the closed loop transfer function that relates y to n.
(c) Sketch the magnitude of the Bode plot associated with the closed loop transfer function that relates y to d.



4. The discrete model for a system has the form, $Y(z)=P(z)U(z)$, where, $P(z)=\dfrac{\beta z^{-1}}{1-\alpha z^{-1}}$.

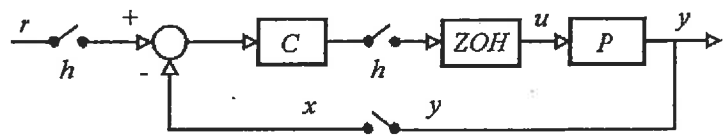
(a) Measurements of $u(k)$ and $y(k)$ are taken at time instants, k, as listed in the Table. Find a least squares estimate for α and β.

k	$y(k)$	$u(k)$
0	100	250
1	129	100
2	125	50
3	108	0

(b) $P(z)$ as noted above, is obtained as the zero order hold equivalent of a continuous time transfer function, $P(s)$. Determine $P(s)$ assuming the sample period is $h=1$ s.

5. Consider the sampled data and digital control system to the right. The input to the ZOH and (continuous) output, y, are uniformly sampled with a sample period of h. $C(z)$ and $P(s)$ *are* given by,

$$C(z) = Kz^{-1}, \quad P(s) = \frac{1}{s}$$



(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $X(z)$ to $R(z)$.

(b) Sketch and annotate the root locus as K varies from zero to infinity.

(c) Is the closed loop system stable for all values of K? If not determine the limiting value of K for stability.

(d) Assume $Kh = 0.5$, and r is initially zero up until $t = 0$, with all initial conditions zero. Suddenly r changes as indicated below.

$$r(0) = 1, r(h) = 0, r(2h) = 0, r(3h) = 0, r(4h) = 0$$

Sketch and carefully annotate the transient response at $y(t)$ for $0 \le t \le 4h$.

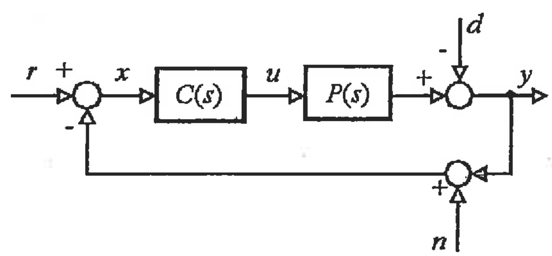
6. Consider the feedback system below with, $C(s) = K, \quad P(s) = \dfrac{0.8e^{-s}}{s}$.

(a) Determine the gain and phase margin when $K = 1$.

(b) Determine the value of K that results in a gain margin of 6db.

(c) Using the value of K from Part (b), determine the steady state value of x when d is a unit step, $r = 0$, and $n = 0$.

(d) Using the value of K from Part (b), determine the steady state value of y when $d = 0$, $r = \sin 3t$, and $n = 0$.



Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{(C+jD)z}{z-re^{j\varphi}}+\dfrac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{(z-a)^{r}}$, $r=2,3\ldots$	$\dfrac{Kn(n-1)\ldots(n-r+2)}{(r-1)!a^{r-1}}a^{n}$

Table of Laplace and z-Transforms
(h denotes the sample period)

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$