Professional Engineers of Ontario

Annual Examinations – May 2012

07-Elec-B3
Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. A Casio or Sharp approved calculator is permitted.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(25 marks) 1. This question concerns error-control coding.

(5 marks) a. Suppose a convolutional encoder has generator polynomials

$$g_1(D) = 1 + D^2$$
$$g_2(D) = 1 + D + D^2$$

For each input, the outputs are read out as g_1 first, then g_2. If the input to the convolutional encoder is 10110, the initial state is all-zero, and the encoder uses zero padding, give the encoded output.

(20 marks) b. For the same convolutional encoder, suppose the receiver observes 00111110000111. Assuming the encoder starts and ends in the all-zero state, use the Viterbi algorithm to determine the most likely input to the encoder, correcting any errors.

(25 marks) 2. This question considers link budgeting.

(10 marks) a. Consider a wireless system with transmitter power of 1 W, antenna gains of 0 dB, receiver losses of 3 dB, receiver noise figure of -168 dBm/Hz, a bandwidth of 10 MHz, and a fading margin requirement of 6 dB. Aside from free-space losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 6 dB, what is the maximum allowed path loss (in dB)?

(10 marks) b. Using a free-space path loss of $20 \log_{10}(4\pi df/c)$, where d represents the distance from transmitter to receiver, f represents the carrier frequency, and c represents the speed of light ($c = 3.0\times10^8$ m/s), and assuming a carrier frequency of 2.4 GHz, find the maximum distance between transmitter and receiver given the system in part a.

(5 marks) c. Discuss the role of the path loss exponent in modifying the free-space path loss in part b.

(25 marks) 3. This question concerns sampling and D/A conversion.

(5 marks) a. The human ear can hear signals up to 20 kHz. What is the Nyquist sampling rate that allows exact reproduction of sounds audible to the ear?

(10 marks) b. Briefly explain pulse code modulation (PCM). If PCM is used to encode the audio signal from part a with 16 bits per sample, what is the required data rate to represent the signal?

(5 marks) c. Suppose PCM is used to sample a signal restricted between -10 V and +10 V. The maximum allowed quantization error is 0.04 V. How many bits per sample are required?

(5 marks) d. Considering your answer for part b, modern audio encoding standards (e.g., MP3, AAC) use data rates of around 128 kbits/s. Give one explanation for any similarity or significant difference in rates.

(25 marks) 4. This question concerns signal modulation and detection.

(5 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_0(t) = \begin{cases} 1, & 0 \leq t \leq T; \\ 0 & \text{elsewhere} \end{cases}$$

and $s_1(t) = - s_0(t)$. Sketch the two signals, and sketch the impulse response of the matched filter $m(t)$, assuming the filter is matched to $s_0(t)$, and assuming the filter output is sampled at time T.

(5 marks) b. Sketch the convolution of $s_0(t)$ and $m(t)$ as a function of t.

(5 marks) c. At the sampling instant (time T), the matched filter output is corrupted by additive Gaussian noise with zero mean and variance σ^2. Give the optimal decision rule assuming that 0 and 1 are equiprobable.

(10 marks) d. Given that

$$\frac{1}{2}\mathrm{erfc}\left(\frac{t-\mu}{\sqrt{2\sigma^2}}\right) = \int_t^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

and given your decision rule from part c, express the probability of error *given than a 1 was sent* in terms of erfc.

(25 marks) 5. This question concerns source coding.

(5 marks) a. Suppose you have a source with five letters: A, B, C, D, E, and F; with probabilities given by Pr(A)=0.16, Pr(B)=0.22, Pr(C)=0.37, Pr(D)=0.11, Pr(E)=0.09, Pr(F)=0.05. Calculate the entropy of this source.

(10 marks) b. Obtain a Huffman code for the source in part a, and calculate its average length.

(5 marks) c. Consider the following code for the source in part a: A=11, B=0, C=1, D=10, E=01, F=00. Is this code a good alternative to the code from part b? Briefly explain.

(5 marks) d. Is it possible for your answer in part b to be less than the answer in part a? Briefly explain.