

PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – May 2012

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) made with the answer.
2. This examination paper is open book; candidates are permitted to make use of any textbooks, references or notes.
3. Any non-communicating calculator is permitted. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.
4. Candidates are required to attempt any five questions. The questions are to be solved within the context of the finite element method.
5. The questions are equally weighted. Indicate which five questions are to be marked on the cover of the first examination workbook.

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**Question 1.** [20 marks] A field variable  $f(x, y) = x^3 y$  is defined over a rectangular domain  $\Omega = \{ \mathbb{R}^+ : 0 \leq x \leq 4, 0 \leq y \leq 6 \}$ . Given the expression

$$g = \int_0^6 \int_0^4 x^3 y \, dx dy$$

and assume the following bilinear interpolation shape functions are used to discretize the spatial/geometric variables  $x$  and  $y$ :

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta), N_2 = \frac{1}{4}(1+\xi)(1-\eta), N_3 = \frac{1}{4}(1+\xi)(1+\eta), N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

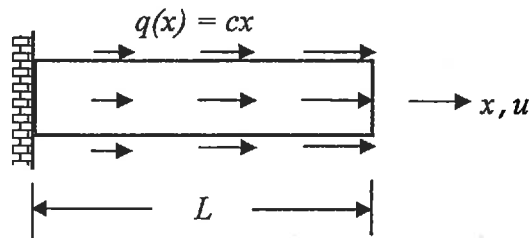
where  $-1 \leq \xi, \eta \leq 1$  for the local coordinates  $\xi, \eta$ .

(a) [15 marks] Determine the value of  $g$  using the Gauss quadrature numerical integration method.

(b) [5 marks] Explain any similarity or difference between your answer and the exact solution  $g = 1152$ .

**Question 2.** [20 marks] A cantilevered bar is loaded by a linearly varying distributed load

$q(x) = cx$  as shown in the figure - note that  $c$  is a constant. The cross-sectional area and length of the bar are denoted by  $A$  and  $L$ , respectively, and it is made of a material with Young's modulus of elasticity  $E$ . The system governing equation can be written as

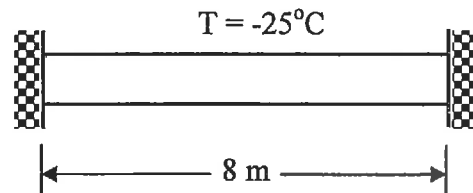


$$EA \frac{d^2 u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\text{subject to: } u(0) = 0 \quad \text{and} \quad EA \frac{du(x)}{dx} \Big|_{x=L} = 0$$

Use the Collocation method to determine an approximate cubic polynomial solution with evaluation points at  $x = \frac{1}{3}L$  and  $x = \frac{2}{3}L$ .

**Question 3.** [20 marks] A one-dimensional steel bar fixed at each end is shown in the figure. The properties of the bar are: Young's modulus  $E = 210$  GPa, cross-sectional area  $A = 12 \times 10^{-3} \text{ m}^2$ , and coefficient of thermal expansion  $\alpha = 12 \times 10^{-6} (\text{mm/mm})/^\circ\text{C}$ . If the bar is subjected to a uniform temperature drop of  $T = -25^\circ\text{C}$  as shown,



(a) [17 marks] Determine the reactions at the fixed ends and the stress in the bar.

(b) [3 marks] Why is the stress in the bar compressive or tensile?

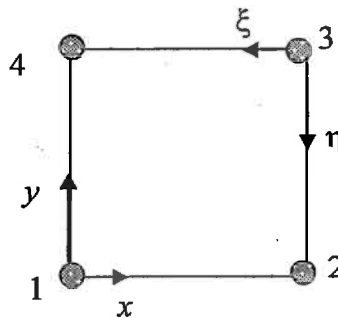
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## Question 4. [20 marks]

- (a) [4 marks] Briefly explain the meaning of geometric isotropy in a sentence or two.
- (b) [6 marks] State the two properties that a polynomial representation of a field variable variation in an element should have to ensure that the element has geometric isotropy?
- (c) [10 marks] Consider the square element below for which the field variable  $u$  is interpolated in the Cartesian  $x, y$  coordinate axes attached at node 1 as

$$u(x, y) = C_1 + C_2x + C_3y + C_4xy$$

Assuming the length of each side of the element is  $L$ , use the  $\xi, \eta$  coordinate axes centred at node 3 to show that the element has geometric isotropy.

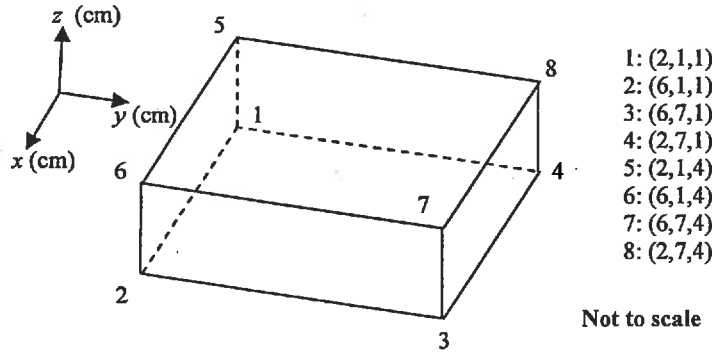


## Question 5. [20 marks]

- (a) [3 marks] Briefly explain in a sentence or two the difference between basis function and shape function.
- (b) [3 marks] Briefly explain in a sentence or two why finite element solutions improve with increasing number of elements.
- (c) [4 marks] Briefly explain in a sentence or two when a discretization implemented using Bubnov-Galerkin method (i.e., Galerkin method) is identical to that obtained using Ritz method.
- (d) [4 marks] Briefly explain in a sentence or two the concepts of  $h$ -element and  $p$ -element in the adaptive finite element method.
- (e) [6 marks] A solid mechanics analyst observed a very slow convergence while using an in-house developed finite element computer code to implement static and modal analyses of a Timoshenko beam. On closer examination of the field variables, the analyst noticed that the transverse displacement was interpolated with a quadratic function and rotation by a linear function. What phenomenon is the analyst witnessing and how can the problem be resolved?

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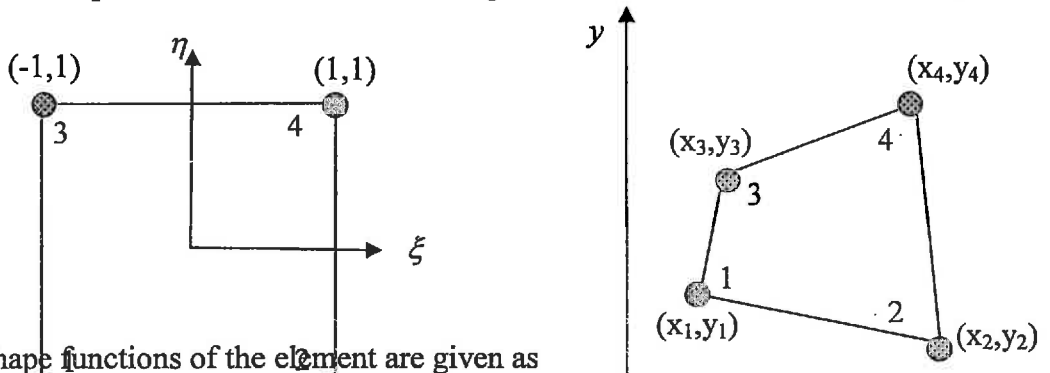
Question 6. [20 marks]



- (a) [4 marks] Determine the shape functions ( $N_i$ ,  $i = 1$  to 8) of an eight-node hexahedron element in natural/parametric coordinates  $(\xi, \eta, \zeta)$  such that  $-1 \leq \xi, \eta, \zeta \leq 1$ . The node numbering is identical to that shown in the above representative global element.
- (b) [3 marks] Evaluate the shape function  $N_3$  at the third node, the fifth node, and the centroid of the element.
- (c) [3 marks] Assume the field variables to the problem are displacements denoted by  $u$ ,  $v$ , and  $w$ . If the only non-zero nodal displacements are  $u_2 = u_3 = u_6 = u_7 = 0.05$  mm, compute the field variables in the natural/ local coordinates  $(\xi, \eta, \zeta)$ .
- (d) [7 marks] Determine the Jacobian matrix and evaluate the Jacobian of the above element.
- (e) [3 marks] Determine the normal strain  $\epsilon_x = \frac{\partial u}{\partial x}$  and shear strain  $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ .

Question 7. [20 marks]

- (a) [2 marks] What is an isoparametric element?
- (b) [10 marks] The four-node isoparametric quadrilateral element shown below is used to map a region in the parent domain into that in the global domain.



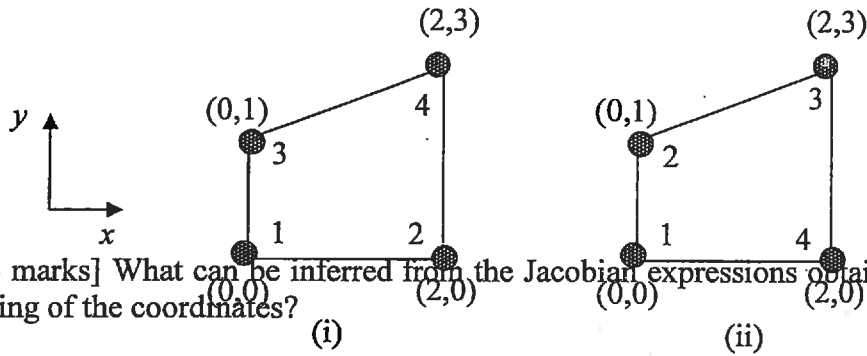
The shape functions of the element are given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta), N_2 = \frac{1}{4}(1+\xi)(1-\eta), N_3 = \frac{1}{4}(1-\xi)(1+\eta), N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

Determine the Jacobian matrix of the element.

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(c) [4 marks] Use the Jacobian matrix obtained in (b) to evaluate the Jacobian of the following elements.



(d) [4 marks] What can be inferred from the Jacobian expressions obtained in (c) in light of the mapping of the coordinates?

(i)

(ii)