National Exams May 2012

07-Mec-B6, Advanced Fluid Mechanics

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any non-communicating calculator. The exam is OPEN BOOK.
3. Answer any <u>3</u> of the 4 questions in Part **A** and any <u>2</u> of the 3 questions in Part **B**
4. Weighting: Each question is equally weighted within a section.
   Part **A:** 40%; Part **B** 60%

Part A: Answer any **3** of the following 4 questions.

Question A1: Water (ρ = 1000 kg/m^3; μ= 0.001 Pa-s) flows at 0.3m/s past a flat plate of length L = 1m and 0.5m wide. On one side, the plate develops naturally (i.e. is laminar) while on the other side it is tripped (using a small wire) such that it is turbulent over the entire length.

a) Evaluate the boundary layer, displacement and momentum thickness at the end of the plate on both sides.
b) Determine the shear stress at the end of the plate and the average shear stress on both sides of the plate.
c) What is the total drag on the plate?

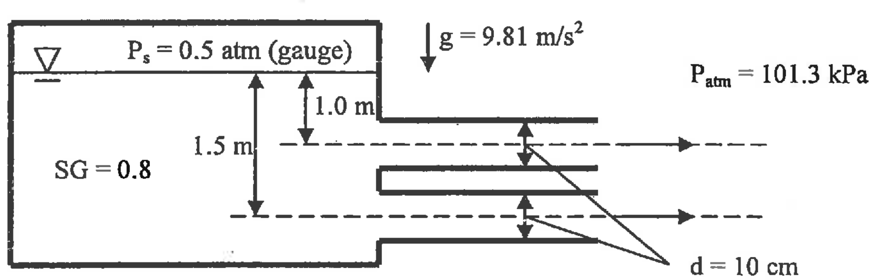
Question A2: An explosion occurs which generates a shock wave travelling through still air (P = 101.3 kPa, T = 293 K). At a distance of 50m from the explosion source, the shock is travelling at 1000 m/s. Assuming that at this distance the radius of curvature is large (i.e. the shock can be approximated as normal), estimate:

a) The static pressure and temperature directly behind the shock.
b) The velocity of the air directly behind the shock.

For air: R = 287 J/kg-K; γ = 1.4.

Question A3: A centrifugal pump, having four stages in parallel, delivers 218 litres/s of liquid against a head of 26 m, the diameter of the impellers being 229 mm and the speed 1,700 rpm. A pump is to be made up with a number of identical stages in series of similar construction to those in the first pump. The new pump is to run at 1,250 rpm, delivering 282 litres/s against a head of 265 m. Find the diameter of the impellers and the number of stages required.

Question A4: An incompressible fluid (a liquid of specific gravity SG = 0.8) is contained in a pressurized reservoir as shown in the figure below. The pressure at the surface of the liquid is 0.5 atm (gauge). The reservoir is connected to two pipes of diameter 10 cm each. Assuming that the reservoir is very large and that free jets occur at the exit (i.e. uniform velocity and constant pressure across the jets), what is the flow rate out of each pipe? Neglect all frictional losses.



Part B: Answer any **2** of the following 3 questions.

Question B1: Two anemometers placed symmetrically about the centre-line of an airport runway at the end of the runway measure the vertical component of velocity. The anemometers are located 100m apart (50m on either side of the runway centre) and measure the velocity 2m above the ground. After the passage of a large aircraft 50m above the runway, the vertical component of velocity measured by the anemometers is 0.5m/s (upwards) 30 seconds after the passage of the aircraft. The velocity measured by these anemometers are due to the wingtip vortices given off by the aircraft (wingspan is 70m). The vortices can be modelled as potential vortices. Their centres coincide with the location of the wing tip when the aircraft passed over the runway end (i.e. 50m above the ground and 70m apart and symmetric about the runway centreline). Calculate the downwash (vertical velocity) experienced by a second, smaller aircraft passing 25m above the runway at the time of the measurement. Verify that the ground corresponds to $v = 0$ and $\psi = \text{constant}$

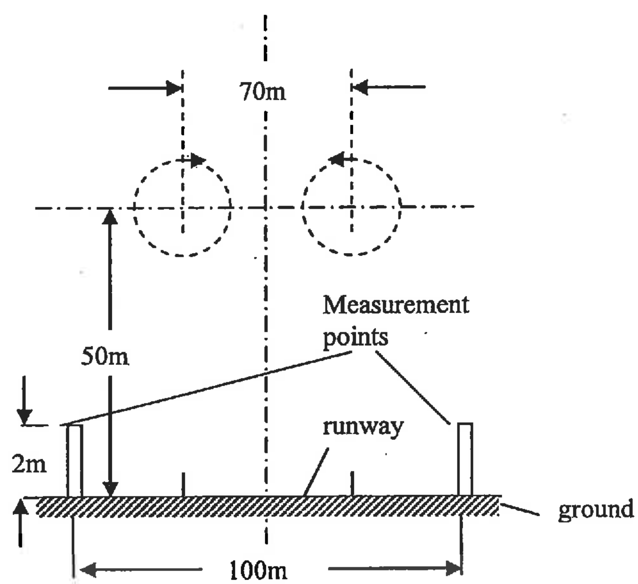


Figure B1: Schematic of trailing vortices over a runway after passage of aircraft.

Question B2: A Newtonian liquid of density ρ and dynamic viscosity μ, flows down through a long cylindrical pipe of constant radius R ($L/R > 100$, L is the length). Assuming that the liquid wets the non-porous pipe walls and that the flow is laminar, derive an expression for the velocity profile in the pipe. Obtain an expression for the flow rate, Q, and the shear stress along the walls. Assume that the pressure gradient along the pipe is zero.

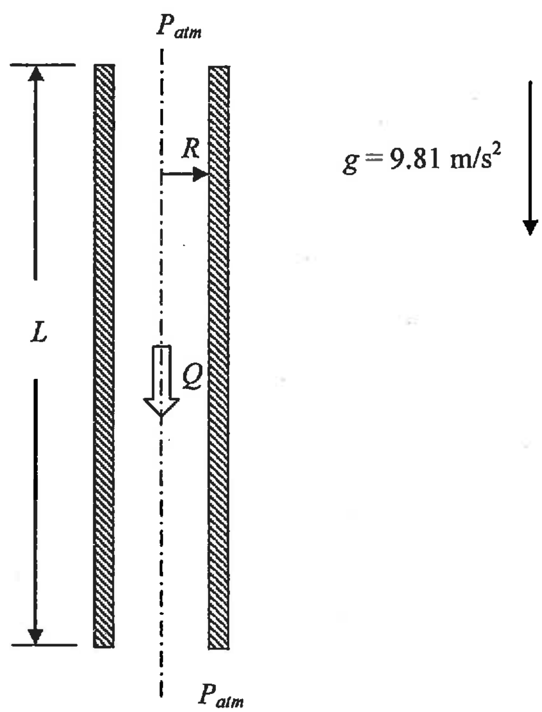


Figure B2: Vertical circular pipe with laminar, steady flow.

Question B3: A constant force of 71N is applied on a plunger pushing a piston of diameter 3cm through an insulted syringe containing air at 20°C ($\gamma = 1.4$, $R = 287 J/kg\text{-}K$). The exit diameter is of 2mm and the ambient atmospheric pressure is $P_b = 101.3kPa$. Estimate the temperature of the air leaving the syringe and the time needed to empty the syringe, given that at the beginning of the stroke the air chamber is 6cm long. The piston moves at a constant rate. Frictional losses may be considered negligible.

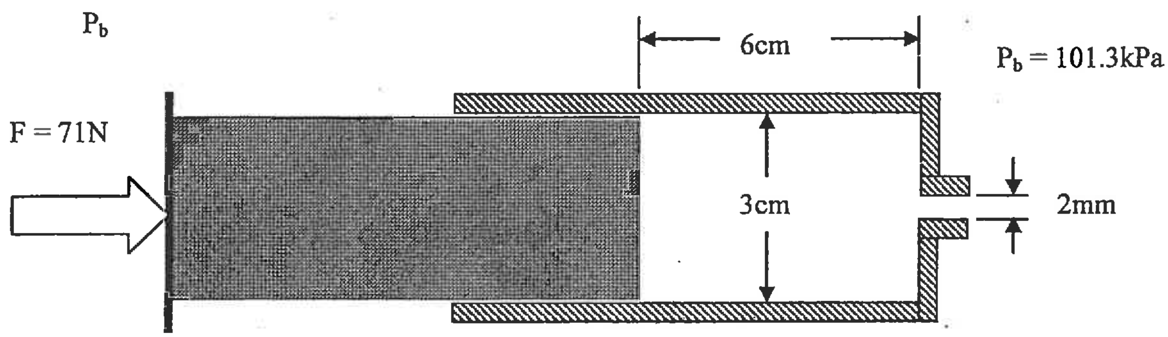


Figure B3: Schematic representation of plunger-syringe arrangement.

Aid Sheets

Compressible Flow:

Adiabatic flow: $$\frac{T_o}{T}=1+\frac{\gamma-1}{2}M^2$$

Isentropic flow: $$\frac{P_o}{P}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad \frac{\rho_o}{\rho}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$$

$$\dot{m}=\rho UA=\sqrt{\frac{\gamma}{R}}\cdot\frac{P_o}{\sqrt{T_o}}\cdot M\cdot\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}A$$

Shock Relations: $$\frac{P_2}{P_1}=\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}=\frac{2\gamma M_1^{\,2}}{\gamma+1}-\frac{\gamma-1}{\gamma+1} \qquad M_2^2=\frac{M_1^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1}$$

Conservation Equations for Cylindrical-polar Coordinate system

$$\vec{U} = v_r \vec{e_r} + v_\theta \vec{e_\theta} + v_z \vec{e_z}$$

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Linear Momentum Equations:

***r*-momentum:**

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right]$$
$$= -\frac{\partial P}{\partial r} + \rho g_r + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{r\theta}) + \frac{\partial}{\partial z}(\tau_{rz}) - \frac{\tau_{\theta\theta}}{r}$$

θ-momentum:

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right]$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \rho g_\theta + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{\theta r}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}) + \frac{\partial}{\partial z}(\tau_{\theta z})$$

***z*-momentum:**

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right]$$
$$= -\frac{\partial P}{\partial z} + \rho g_z + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{zr}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{z\theta}) + \frac{\partial}{\partial z}(\tau_{zz})$$

$$\tau_{rr} = \mu\left(2\frac{\partial v_r}{\partial r} - \frac{2}{3}\nabla\cdot\vec{U}\right) \qquad \tau_{\theta\theta} = \mu\left(2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}\nabla\cdot\vec{U}\right)$$

$$\tau_{zz} = \mu\left(2\frac{\partial v_z}{\partial z} - \frac{2}{3}\nabla\cdot\vec{U}\right) \qquad \tau_{r\theta} = \tau_{\theta r} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$$

$$\tau_{rz} = \tau_{zr} = \mu\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right) \qquad \tau_{\theta z} = \tau_{z\theta} = \mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$$

$$\nabla\cdot\vec{U} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z)$$

$$\nabla\times\vec{U} = \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}\right)\vec{e_r} + \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right)\vec{e_\theta} + \left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)\vec{e_z}$$